Mathematical Institute   Prof. Dr. Wolfgang König
University Leipzig   Dr. Ramon Plaza
Summer term 2005

ODE for Physicists - Homework 1

Due: April 12, 2005

1. (4 pts.) Express the following statements as differential equations.

   (a) For a certain substance the rate of change of vapour pressure
   P with respect to temperature T is proportional to the vapour
   pressure and inversely proportional to the square of temperature.

   (b) The potential difference E across an element of inductance L is
   equal to the product of L and the time rate of change of the
   current I in the inductance.

2. (4 pts.) Find all the curves in the (x, y) plane determined by the
   following conditions.

   (a) The segment joining $P(x, y)$ and the point of intersection of the
   normal at P with the x axis is bisected by the y axis.

   (b) At each point (x, y) the length of the subtangent[1] is equal to the
   sum of the coordinates of the point.

3. (4 pts.) Draw direction fields and some solution curves (at $y = -1$,
   $-1/2$, 0, 1) for the ODEs:

   (a) $y' = x^2 + y^2 - 1$,
   (b) $y' = 1 - y^2$.

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[1]The *subtangent* is the projection of the tangent upon the axis of abscissas (i.e., the
x-axis). "Tangent" here specifically means a line segment which is tangential to a point
P on a curve and which intersects the x-axis at point Q. The line segment PQ is the
tangent, and the length of PQ is also called the *tangent*. Draw a line through P parallel
to the axis of ordinates (i.e., the y-axis). This line intersects the x-axis at P'. Then line
$P'Q$ is the *subtangent*, and its length is also called the subtangent.