Berlin Leipzig Seminar

Analysis/probability theory

First Meeting Winter Term 2011/12

Organized by the DFG Research Group *Analysis and Stochastics in Complex Physical Systems*

DATE: Friday, 4 November, 2011

VENUE: TU Braunschweig, Neuer Senatssaal, Altgebäude, Pockelsstr. 4, 38106 Braunschweig

PROGRAMME:

10:00–10:50: **Miguel Ballesteros (TU Braunschweig)**

Existence and construction of resonances for atoms coupled to the quantized radiation field

Abstract: The theory of quantum mechanics asserts that the energy of an atom can be quantized. The possible energies are the eigenvalues of the Schrödinger equation. The lowest eigenvalue is called the ground state energy and the other eigenvalues are the excited energies. The Schrödinger equation predicts that if an atom is in an excited state at some time then it remains in that state forever. Experiments show, however, that the atom does not remain in an excited state forever, but decays to a lower energy state. During this decay, the atom emits photons whose energy is given by the difference between the initial and the final energy, in accordance with the Bohr's frequency condition. The process described above can be expressed on a technical level as follows: the atom remains in a certain state for a short period and then decays to a lower energy state. Thus the excited states are not eigenvalues of a certain Hamiltonian but turn into *resonances*. These resonances appear when the photon field is introduced into the picture. We analyze the Pauli-Fierz model, which represents a non-relativistic atom coupled to a (quantized) photon field. We prove that the excited eigenvalues of the atom give rise to resonances, once the photon field is introduced, and that the energies of the resonance-producing photons are given by Bohr's frequency condition, up to second order in the coupling constant. We do not assume that there is an infrared regularization but we require an ultraviolet cutoff. We review Sigal's recent construction of resonances based on renormalization group analysis and present a novel alternative construction based on "Pizzo's Method". This is a joint work with Volker Bach, Alessandro Pizzo and Marwan Shoufan.

11:00–11:50: **Jonas Tölle (TU Berlin)**

Singular stochastic evolution inclusions and ergodicity

Abstract: We provide an abstract variational existence and uniqueness result for multi-valued monotone non-coercive stochastic evolution inclusions in Hilbert spaces with general additive noise and Gaussian multiplicative noise.

In particular, we consider stochastic diffusion inclusions of the type

$$\begin{cases} dX_t \in \left\{ \begin{array}{l} \mathrm{div}[\phi(\nabla X_t)] \\ L[\psi(X_t)] \end{array} \right\} dt + \left\{ \begin{array}{l} dN_t \\ B_t(X_t)\, dW_t \end{array} \right. \\ \qquad\qquad\qquad X_0 = x, \end{cases} \tag{1}$$

where $\phi \subset \mathbb{R}^d \times \mathbb{R}^d$, $\psi \subset \mathbb{R} \times \mathbb{R}$ are cyclically monotone graphs with sublinear growth (possibly the Heaviside function), L is a dissipative Markov operator, $\{N_t\}$ is a general space-regular càdlàg process, $\{B_t(\cdot)\}$ is a family of Hilbert-Schmidt operators, $\{W_t\}$ is a cylindrical Wiener process. The initial condition x is chosen either in L^2 or some negative order Sobolev space associated to L.

The solutions to (1) depend continuously on ϕ and ψ with respect to convergence of graphs (that is, pointwise convergence of their primitives).

For equations (1), in the case of additive Wiener noise, we prove the existence of a unique invariant measure (for all space dimensions) which is weak* mean ergodic if we assume suitable Sobolev embeddings.

Applications to random dynamical systems for noise with stationary increments are also discussed.

(Joint work with Benjamin Gess, Bielefeld)

12:00–12:50: **Georg Weiss (Universität Düsseldorf)**

Pulsating waves in self-propagating high temperature synthesis

Abstract: We derive the precise limit of SHS in the high activation energy scaling suggested by B.J. Matkowksy-G.I. Sivashinsky in 1978 and by A. Bayliss – B.J. Matkowksy – A.P. Aldushin in 2002. In the time-increasing case the limit coincides with the Stefan problem for supercooled water with spatially inhomogeneous coefficients. In general it is a nonlinear forward-backward parabolic equation with discontinuous hysteresis term.

In the first part of the talk we give a complete characterization of the limit problem in the case of one space dimension. In the second part we construct in any finite dimension a rather large family of pulsating waves for the limit problem. In the third part, we prove that for constant coefficients the limit problem in any finite dimension does not admit non-trivial pulsating waves. The combination of all three parts strongly suggests a relation between the pulsating waves constructed and the numerically observed pulsating waves for finite activation energy in dimension $n \geq 1$ and therefore provides a possible and surprising explanation for the phenomena observed. (joint work with Regis Monneau, CERMICS)

Everybody is welcome to attend. Wolfgang König, TU Berlin and WIAS Berlin