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Peter Friz · Wolfgang König ·
Chiranjib Mukherjee · Stefano Olla
Editors

Probability and Analysis in Interacting Physical Systems

In Honor of S.R.S. Varadhan,
Berlin, August, 2016

 Springer

Editors

Peter Friz
Institute of Mathematics
Technische Universität Berlin
Berlin, Germany

Wolfgang König
Weierstrass Institute for Applied
Analysis and Stochastics
Berlin, Germany

Chiranjib Mukherjee
Institute for Mathematical Stochastics
University of Münster
Münster, Germany

Stefano Olla 
CEREMADE, UMR CNRS 7534
Université Paris Dauphine - PSL
Paris, France

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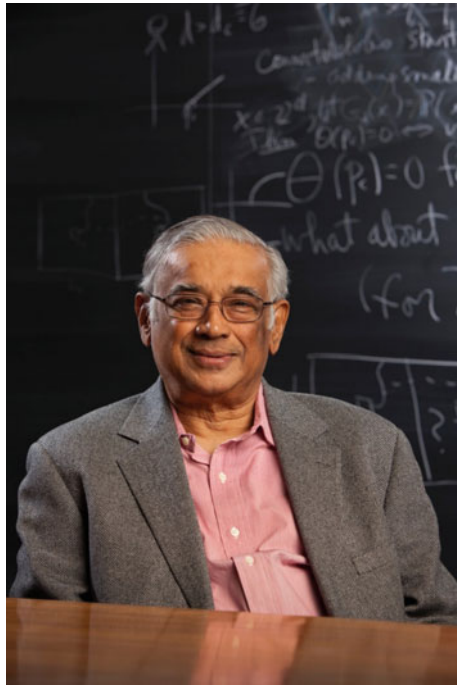
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Preface

This festschrift marks the occasion of the 75th birthday of S. R. S. Varadhan, one of the most influential researchers in the field of probability for the last 50 years. This volume contains ten research articles authored by several of Varadhan's former PhD students and/or close collaborators. The topics of the papers are more or less closely linked with some of his deepest interests over the decades: large deviations, Markov processes, interacting particle systems, motions in random media and homogenisation, reaction–diffusion equations and directed last-passage percolation. This diverse span of subjects illustrates the wide range of Varadhan's research.

S. R. S. Varadhan was a pioneer in many important developments in the understanding of the asymptotic behaviour of random processes, such as Brownian motions, or interacting particle systems, in random and in non-random environments. Many of his contributions are fundamental and have attracted entire generations of researchers, giving them material to work on for decades. Consider, for example, the idea of martingale characterisations of Markov processes that he developed with Daniel Stroock around 1970, the series of papers from the early 1970s with Monroe Donsker that laid the foundations of a large-deviation analysis of exponential functionals of Markov chains, the work on homogenisation with George Papanicolaou and the joint paper with Claude that introduced a method for proving central limit theorems for motions in random media (which is still the basis of most research in this area), his work on hydrodynamic limits of large interacting particle systems, and many more groundbreaking developments that he initiated with his co-authors.

His worldwide esteem has been enormous over the decades, and he has been the recipient of a number of prestigious prizes, of which we mention here only the Abel Prize (2007) for his *fundamental contributions to the theory of probabilities, in particular the creation of a unified theory of large deviations*.

S. R. S. Varadhan has inspired and motivated many young gifted students, and he has attracted a great number of strong early-career PhD researchers. This is reflected in the total of 37 successfully finished PhD projects over the decades. Furthermore, a large percentage of these students have developed respectable

academic careers themselves, spreading Varadhan's ideas and favourite subjects to their own PhD students and colleagues.

Let us give a short survey of the scientific content of this festschrift.

Chatterjee introduces and surveys the probabilistic theory and open questions of the Euclidean version of the Yang–Mills theory and corresponding lattice gauge theories, in particular their continuum limits. He formulates in probabilistic terms the questions that theoretical physicists ask and gives a brief survey of the probabilistic literature.

Chevyrev, Friz, Korepanov, Melbourne and Zhang review the origins of the convergence of fast-slow deterministic systems to stochastic differential equations and revisit and improve a proof of Kelly and Melbourne using recent progress on p -variation and càdlàg rough-path analysis.

Kosioris, Loulakis and Souganidis study the shallow lake problem from economics and identify the welfare function as a viscosity solution of the associated Bellman equation. They then derive several properties of the solution, including its asymptotic behaviour at infinity, and conclude with a numerical scheme.

Joseph, Rassoul-Agha and Seppäläinen study the motion of independent particles in a certain kind of dynamical random environment in the d -dimensional discrete space, where the distribution of the environment has a product structure. They characterize the class of spatially ergodic invariant measures, study their correlation structure and draw conclusions about the convergence of the particle distribution to equilibrium in dimensions one and two.

Reaction–diffusion equations, more precisely, the heat equation in random (here Weibull distributed) potential, are considered by Ben Arous, Molchanov and Ramirez; they concentrate on approximation in boxes that are so large that the mean over them is a kind of interpolation between the moment asymptotics (ergodic theorem) and the quenched (i.e., almost-sure) asymptotics; stable limiting distributions are obtained.

Bröker and Mukherjee consider a mollified version of the stochastic heat equation (random Brownian polymer in time-space white noise) in dimension ≥ 3 and prove the convergence of the rescaled polymer in distribution to a Gaussian distribution.

Bisi and Zygouras consider point-to-line and point-to-half-line directed last-passage percolation with exponentially distributed waiting times. They derive Sasamoto's Fredholm determinant formula for the Tracy–Widom GOE distribution and the one-point marginal distribution of the Airy $_{2\rightarrow 1}$ process, which was originally derived by Borodin, Ferrari and Sasamoto.

Landim, Chang and Lee prove an energy estimate for the polar empirical measure of the two-dimensional symmetric simple exclusion process. They deduce from this estimate, and from their earlier results, large deviations principles for the polar empirical measure and for the occupation time of the origin.

Sethuraman and Venkataramani consider a time-dependent growing random-graph model of preferential-attachment type, where new nodes are attached to existing ones according to some superlinear function of their degrees. From earlier work, the emergence of condensation is known. Here, they establish laws of

large number and fluctuation results for the number of nodes at a given time with a given degree and recover the emergence of the condensate in greater detail.

Pinsky considers the distribution of a certain random polynomial of order N of the prime numbers, whose powers are independent geometric random variables with parameter equal to one minus the reciprocal of the basis (the prime). He shows that the logarithm of this random quantity, when divided by $\log N$, converges in distribution to the Buchstab distribution. As a corollary, Merten's theorem from multiplicative number theory is recovered.

This festschrift grew out of a birthday workshop on the occasion of Varadhan's 75th birthday, which took place at TU Berlin on 15–19 August 2016. It was a great honour and pleasure for us to organize this event.

We wish you, dear Raghu, many further years of much joy of doing mathematical research and a most stable health to carry through all your plans that you have!

Berlin, Münster and Rome
December 2018

Peter Friz
Wolfgang König
Chiranjib Mukherjee
Stefano Olla

List of Participants of the workshop at TU Berlin, 15–19 August 2016

Inez Armendariz, Universidad de Buenos Aires, Buenos Aires, Argentina
Sigurd Assing, University of Warwick, Coventry, UK
Peter Bank, TU Berlin, Berlin, Germany
Christian Bayer, WIAS Berlin, Berlin, Germany
Gérard Ben Arous, Courant Institute, New York, New York, USA
Erwin Bolthausen, University of Zurich, Zurich, Switzerland
Yvain Bruned, University of Warwick, Coventry, UK
Catriona Byrne, Springer, Heidelberg, Germany
Jiawei Cheng, University of Oxford, Oxford, UK
Ilya Chevyrev, TU Berlin, Berlin, Germany
Alessandra Cipriani, WIAS Berlin, Berlin, Germany
David Criens, TU München München, Germany
Apostolos Damialis, De Gruyter, Berlin, Germany
Jean Dominique, Deuschel, TU Berlin, Berlin, Germany
Joscha Diehl, Max-Planck-Institut, Leipzig, Germany
Alexander Drewitz, Universität zu Köln, Köln, Germany
Leif Döring, Universität Mannheim, Mannheim, Germany
Peter Eichelsbacher, Universität Bochum, Bochum, Germany
Shima Elesaely, TU Berlin, BMS, Berlin, Germany
Benjamin Fehrman, Max-Planck-Institut, Leipzig, Germany
Franziska Flegel, WIAS Berlin, Berlin, Germany
Klaus Fleischmann, WIAS Berlin, Berlin, Germany
Peter Friz, WIAS Berlin/TU Berlin, Berlin, Germany
Nina Gantert, TU München, München, Germany
Adrian Gonzalez Casanova, WIAS Berlin, Berlin, Germany
Jürgen Gärtner, TU Berlin, Berlin, Germany
Onur Gün, WIAS Berlin, Berlin, Germany
Matthias Hammer, TU Berlin, Berlin, Germany
Martin Hairer, University of Warwick, Warwick, UK
Christian Hirsch, WIAS Berlin, Berlin, Germany
Antoine Hocquet, TU Berlin, Berlin, Germany

Antoine Jacquier, Imperial College London, London, UK
Benedikt Jahnel, WIAS Berlin, Berlin, Germany
Marvin Kettner, TU Darmstadt, Darmstadt, Germany
Frederik Klement, Universität Mainz, Mainz, Germany
Tom Klose, HU Berlin, Berlin, Germany
Elena Kosygina, Baruch College/CUNY, Graduate Center, New York, New York, USA
Richard Kraaij, TU Delft, Delft, The Netherlands
Christof Külske, Universität Bochum, Bochum, Germany
Wolfgang König, WIAS Berlin/TU Berlin, Berlin, Germany
Claudio Landim, IMPA, Rio de Janeiro, Brasilien
Michail Loulakis, National Technical University of Athens, Athens, Greece
Hoang Duc Luu, Max-Planck-Institut, Leipzig, Germany
Terry Lyons, Oxford University, Oxford, UK
Matthias Löwe, Universität Münster, Münster, Germany
Vlad Margarint, University of Oxford, Oxford, UK
Mario Maurelli, WIAS Berlin/TU Berlin, Berlin, Germany
Georg Menz, University of California, Los Angeles, California, USA
Markus Mittnenzweig, WIAS Berlin, Berlin, Germany
Marvin Mueller, ETH Zurich, Zurich, Switzerland
Chiranjib Mukherjee, Courant Institute, New York, New York, USA/WIAS Berlin, Berlin, Germany
Christian Mönch, TU Darmstadt, Darmstadt, Germany
Jan Nagel, TU München, München, Germany
Sina Nerjad, University of Oxford, Oxford, UK
Stefano Olla, Université Paris Dauphine, Paris, France
Nicolas Perkowski, HU Berlin, Berlin, Germany
Ross Pinsky, Technion-Israel Institute of Technology, Haifa, Israel
David Prömel, ETH Zurich, Zurich, Switzerland
Alejandro Ramírez, Pontificia Universidad Católica de Chile, Santiago, Chile
Jose Ramírez, Universidad de Costa Rica, San Pedro, Costa Rica
Firas Rassoul-Agha, University of Utah, Salt Lake City, Utah, USA
Martin Redmann, WIAS Berlin, Berlin, Germany
Max von Renesse, Universität Leipzig, Leipzig, Germany
Michiel Renger, WIAS Berlin, Berlin, Germany
Fraydoun Rezakhanlou, University of California, Berkeley, California, USA
Sebastian Riedel, TU Berlin, Berlin, Germany
Michael Röckner, Universität Bielefeld, Bielefeld, Germany
Renato Soares dos Santos, WIAS Berlin, Berlin, Germany
Alexandros Saplaouras, TU Berlin, Berlin, Germany
Michael Scheutzow, TU Berlin, Berlin, Germany
Massimo Secci, TU Berlin, Berlin, Germany
Vitalii Senin, TU Berlin, Berlin, Germany
Insuk Seo, University of California, Berkeley, California, USA
Sunder Sethuraman, University of Arizona, Tucson, Arizona, USA

Atul Shekhar, Indian Statistical Institute, Bangalore, India
Sergio Simonella, TU München, München, Germany
Martin Slowik, TU Berlin, Berlin, Germany
Herbert Spohn, TU München, München, Germany
Claudia Strauch, Universität Heidelberg, Heidelberg, Germany
Tat Dat Tran, Max-Planck-Institut, Leipzig, Germany
Srinivasa Varadhan, Courant Institute, New York, USA
Isabell Vorkastner, TU Berlin, Berlin, Germany
Moritz Voss, TU Berlin, Berlin, Germany
Florian Völlering, Universität Münster, Münster, Germany
Stefan Walter, TU Darmstadt, Darmstadt, Germany
Yilin Wang, ETH Zurich, Zurich, Switzerland
Martin Weidner, Imperial College London, London, UK
Heinrich von Weizsäcker, TU Kaiserslautern, Kaiserslautern, Germany
Bo Xia, Université Paris Sud, Paris, France
Danyu Yang, University of Oxford, Oxford, UK
Horng-Tzer Yau, Harvard University, Cambridge, UK
Atilla Yilmaz, Koc University, Istanbul, Turkey
Ofer Zeitouni, Weizmann Institute, Rehovot, Israel
Deng Zhang, Universität Bielefeld, Bielefeld, Germany
Nikos Zygouras, University of Warwick, Coventry, UK

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