SimParTurS – work report 2008–10–02

Volker John and Carina Suciu

Saarland University, Saarbrücken

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1. Shear Slip Mesh Update Method (SSMUM) in 2D

- M. Behr, T. Tezduyar (1999, 2001)
- principle: one layer of mesh cells has to be updated during the rotation
- rotation of a 2D stirrer

Shear Slip Mesh Update Method (SSMUM) in 2D: work done since the last meeting

- Arbitrary Lagrangian Eulerian (ALE) method for Navier-Stokes equation implemented
  - method for handling time-dependent domains
  - combines the two points of view: Lagrange and Euler
  - transformation from the reference domain to the current domain
  - appearance of an additional convective term due to the domain movement
  - in ALE frame NSE look like:

$$\begin{aligned} u_t - Re^{-1}\Delta u + ((u - w)\cdot\nabla)u + \nabla p &= f \quad \text{in} \quad (0,T)\times\Omega(t) \\ \nabla\cdot u &= 0 \quad \text{in} \quad [0,T)\times\Omega(t) \end{aligned}$$

w – movement of the domain (mesh velocity)

Shear Slip Mesh Update Method (SSMUM) in 2D: work done since the last meeting

- change of finite element spaces implemented
  - delete old structures for fespaces, matrices, arrays
  - allocate new structures for fespaces, matrices, arrays
  - fill new matrices, arrays
- interpolation of P_2^{bubble} finite element functions implemented
  - based on geometrical information (coordinates of the degrees of freedom)
  - interpolation error in shear slip layer occurs
  - important to have some interpolation of bubble functions in shear slip layer, defined as averaged values from neighbour mesh cells outside the shear slip layer

Shear Slip Mesh Update Method (SSMUM) in 2D: numerical example

- initial velocity $\mathbf{u} = \mathbf{0}$
- backward Euler method, $\Delta t = 0.001$
- $P_2^{\mathrm{bubble}}/P_1^{disc}$ finite element method, $37984/18816$ degrees of freedom
- Galerkin FEM
- velocity of the stirrer prescribed
- 5 rotations per second
- $Re = 1000$
- flow in time interval [0,1]

Shear Slip Mesh Update Method (SSMUM) in 2D: summary

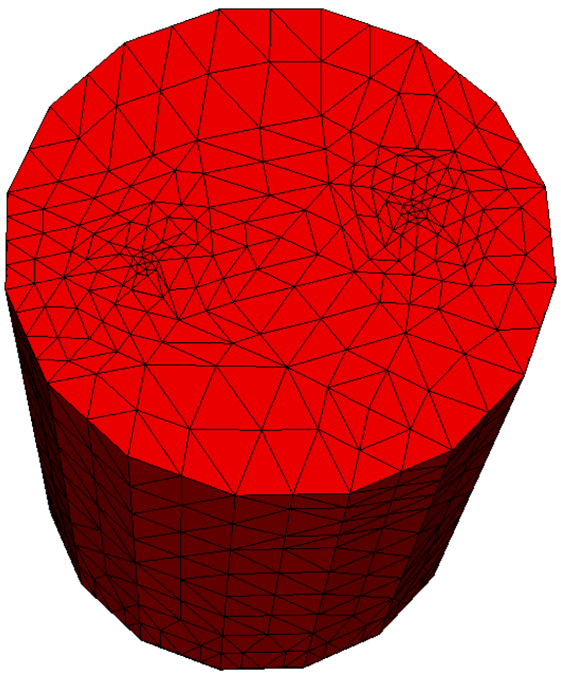
- problem: efficient solver
  - used: GMRES with block Gauss–Seidel preconditioner (Vanka)
  - multigrid methods seem to be unrealistic since after first update of shear slip layer multigrid hierarchy lost

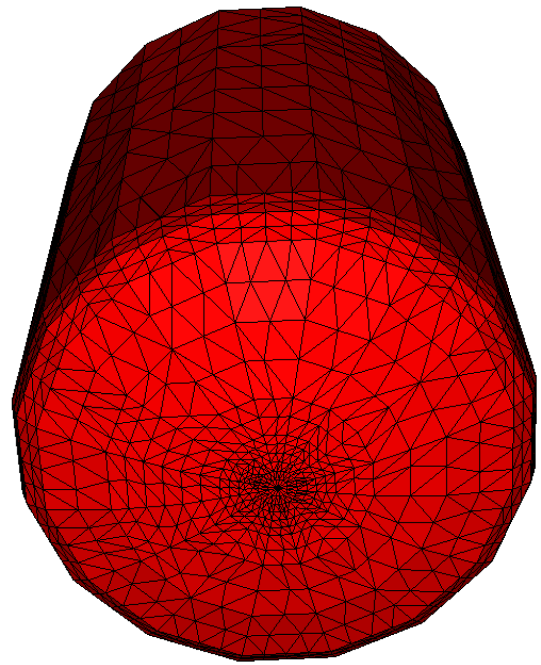
  possible solution: parallelization
- problem: interpolation error of P_2^{bubble} finite element function after mesh update
  - can be seen somewhat in graphical output of results (pressure)
  - $\Longrightarrow$ shear slip layer should be away from regions of interest

  possible solution: using P_1/P_1 finite element (no interpolation error) plus stabilization

2. Flow in a reactor head with an imposed force

- geometry from a PhD thesis MPI Magdeburg
- grid





- initial grid created with TetGen

2. *Flow in a reactor with an imposed force: numerical simulation*

- Crank–Nicolson scheme, $\Delta t = 0.001$
- $P_2^{\text{bubble}}/P_1^{\text{disc}}$, $139\,887/39\,680$ degrees of freedom
- $Re = 10000$
- projection–based Variational Multiscale Method (John,Kaya (2005))
- coarse space: P_0
- turbulent viscosity: Smagorinsky type

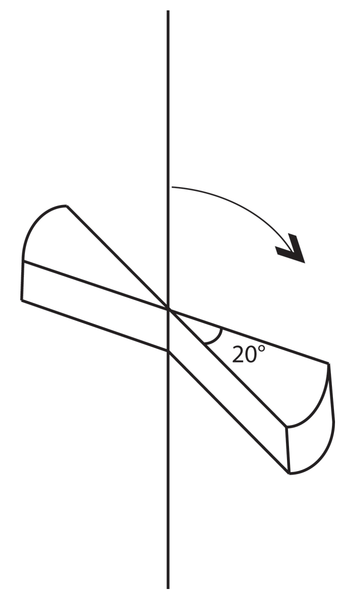
2. Flow in a reactor with an imposed force: numerical simulation

- region of the stirrer: $\alpha = -2\pi t$

$$\Omega_{\text{force}} = \left\{ (x,y,z) \ : \ \sqrt{x^2+y^2} \le 25, \ z \in [65,75], \ \left| \arctan \frac{y}{x} - \alpha \right| < 0.175 \right\}$$

imposed force in region of the stirrer:

$$\mathbf{f} = 2\pi \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix}$$



- flow in time interval [0,5]
- bottom view cutting plane

3. Next steps

- generation of grid with shear slip layer for 3D reactor
  - one layer above stirrer
- extend SSMUM to this situation
  - edge swapping on faces of tetrahedra
- generation of grid with shear slip layers for 3D reactor with stirrer
- coupling of population balance equation (transport equation) to Navier-Stokes equations for reactor domain