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Numerical Mathematics III – Partial Differential Equations

Exercise Problems 10

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Show that the interpolation operator $I_{\hat{K}} \; : \; C^s(\hat{K}) \to \hat{P}(\hat{K})$, which was defined in the lecture, is a linear and continuous operator. In addition, show that the restriction of $I_{\hat{K}}$ to $P(\hat{K})$ is the identity operator.

2. Let the domain $\Omega = (0,1)^2$ be triangulated with the two triangles with the vertices $(0,0),(1,1),(0,1)$ and $(0,0),(1,0),(1,1)$. Compute the error between the function
$$v(x,y) = \sin(2x+y) + 6x^3y^2 \in C^\infty(\Omega)$$
and its interpolant Iv:
   - in the finite element space P_0, the functional is the value of the function in the barycenter of the triangle, the error should be given in the $L^2(\Omega)$ norm,
   - in the finite element space P_1, the functionals are the values of the function in the vertices of the triangles, the error should be given in the $L^2(\Omega)$ norm and in the $H^1(\Omega)$ semi norm.

3. Consider the function
$$v(x) = \begin{cases} 0.5 & x \in (0, 0.25) \\ 1 & x \in (0.25, 0.5) \\ 0 & x \in (0.5, 0.75) \\ -0.5 & x \in (0.75, 1). \end{cases}$$
Compute the Clément interpolant of this function in the finite element space P_1, which is defined on the domain $\Omega = (0,1)$ and the grid with the nodes $\{0, 0.25, 0.5, 0.75, 1\}$.
Hint: the integrals can be computed with Maple, Matematica etc.

4. Let P_1^{nc} the two-dimensional Crouzeix–Raviart finite element space with functions that vanish in the midpoints of the edges which lie on the boundary of the domain. Prove that
$$\|v_h\|_h = \left(\sum_{K \in \mathcal{T}_h} \|\nabla v_h\|^2_{L^2(K)} \right)^{1/2}$$
defines a norm in this space.
Hint: It is clear that $\|v_h\|_h$ defines a semi norm. One has to show that from $\|v_h\|_h = 0$ it follows that $v_h = 0$.

The exercise problems should be solved in groups of two or three students. They have to be submitted until **Tuesday, July 02, 2013** either before or after one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.

These are the last exercise problems.