Lösungen zum 34. Präsenzblatt für MfI 3

1. Aufgabe :

$$
\begin{aligned}
\nabla \times \mathbf{f} &= \begin{pmatrix} \partial_y f_3 - \partial_z f_2 \\ \partial_z f_1 - \partial_x f_3 \\ \partial_x f_2 - \partial_y f_1 \end{pmatrix} \\
&= \begin{pmatrix} 0-0 \\ 0-0 \\ 1-1 \end{pmatrix} \\
&= \mathbf{0} \\
F_x &= 2x + y \\
F &= x^2 + yx + C(y,z) \\
\Longrightarrow \quad F_y &= x + C_y(y,z) \\
&\overset{!}{=} x \\
\Longrightarrow \quad C_y(y,z) &= 0 \\
\Longrightarrow \quad C(y,z) &= D(z) + c \\
F_z &= 0 + D_z(z) \\
&\overset{!}{=} 2z \\
\Longrightarrow \quad D(z) &= z^2 + c \\
\Longrightarrow \quad F(x,y,z) &= x^2 + yx + z^2 + c
\end{aligned}
$$

2. Aufgabe :

$$
\begin{aligned}
f(x,y) &= \frac{1}{y} - \frac{1}{x} - 4x + y \\
\nabla f &= \begin{pmatrix} -4 + x^{-2} \\ 1 - y^{-2} \end{pmatrix} \\
&\overset{!}{=} \mathbf{0} \\
\implies 0 &= -4 + x^{-2} \\
x_{1,2} &= \pm\frac{1}{2} \\
\implies 0 &= 1 - y^{-2} \\
y_{1,2} &= \pm 1 \\
\text{Hesse-Matrix:} & \\
H(f) &= \begin{pmatrix} -2x^{-3} & 0 \\ 0 & 2y^{-3} \end{pmatrix} \\
H(f)(0.5,1) &= \begin{pmatrix} -16 & 0 \\ 0 & 2 \end{pmatrix} \\
\det(H) &< 0 \quad \implies \quad \text{Sattelpunkt} \\
H(f)(0.5,-1) &= \begin{pmatrix} -16 & 0 \\ 0 & -2 \end{pmatrix} \\
\det(H) &> 0 \quad \text{und} \quad f_{xx}, f_{yy} < 0 \quad \implies \quad \text{Maximum} : f(0.5,-1) = -6 \\
H(f)(-0.5,1) &= \begin{pmatrix} 16 & 0 \\ 0 & 2 \end{pmatrix} \\
\det(H) &> 0 \quad \text{und} \quad f_{xx}, f_{yy} > 0 \quad \implies \quad \text{Minimum} : f(-0.5,1) = 6 \\
H(f)(-0.5,-1) &= \begin{pmatrix} 16 & 0 \\ 0 & -2 \end{pmatrix} \\
\det(H) &< 0 \quad \implies \quad \text{Sattelpunkt}
\end{aligned}
$$