

Lösungen zu den 21. Aufgabenblatt für MfI 2

1. Aufgabe :

$$f(x) = \frac{1}{36}x^2 \quad x_1 = 0, \quad x_2 = 12, \quad f(x_1) = 0, \quad f(x_2) = 4$$

Rotation um die x -Achse:

$$\begin{aligned} V_x(x) &= \pi \int_0^{12} \left(\frac{1}{36}x^2 \right)^2 dx \\ &= \frac{1}{1296} \pi \left[\frac{1}{5}x^5 \right]_0^{12} \\ &= \frac{192}{5} \pi \\ &\approx 120,6 \end{aligned}$$

Rotation um die y -Achse:

$$\begin{aligned} V_y(x) &= \pi \int_0^4 (6\sqrt{x})^2 dx \\ &= 36\pi \int_0^4 x dx \\ &= 36\pi \left[\frac{1}{2}x^2 \right]_0^4 \\ &= 288\pi \\ &\approx 904,8 \end{aligned}$$

2. Aufgabe :

Parameterdarstellung:

$$\mathbf{x}(t) = \begin{pmatrix} a \cos^3(t) \\ a \sin^3(t) \end{pmatrix} \implies \dot{\mathbf{x}}(t) = \begin{pmatrix} -3a \cos^2(t) \sin(t) \\ 3a \sin^2(t) \cos(t) \end{pmatrix}$$

$$\begin{aligned} |\dot{\mathbf{x}}(t)| &= \sqrt{(9a^2 \sin^2(t) \cos^4(t)) + (9a^2 \cos^2(t) \sin^4(t))} \\ &= 3a \sqrt{\sin^2(t) \cos^2(t) (\cos^2(t) + \sin^2(t))} \\ &= 3a |\sin(t)| |\cos(t)| \end{aligned}$$

Länge der Astroide:

$$\begin{aligned} l &= \int_0^{2\pi} 3a |\sin(t)| |\cos(t)| \\ &= 4 \int_0^{\frac{\pi}{2}} 3a \sin(t) \cos(t) \\ &= 12a \int_0^{\frac{\pi}{2}} \sin(t) \cos(t) \\ &= 12a \left[-\frac{1}{2} \cos^2(t) \right]_0^{\frac{\pi}{2}} \\ &= 6a \end{aligned}$$

3. Aufgabe :

Mittelpunktsregel:

$$Q(f) = f\left(\frac{a+b}{2}\right) (b-a)$$

Trapezregel:

$$Q(f) = \frac{(b-a)}{2} (f(a) + f(b))$$

Simpson-Regel:

$$Q(f) = \frac{(b-a)}{6} \left(f(a) + 4f\left(\frac{b+a}{2}\right) + f(b) \right)$$

benötigte Funktionswerte:

$$f\left(-\frac{16}{16}\right) = -1.2756$$

$$f\left(-\frac{15}{16}\right) = -1.1177$$

$$f\left(-\frac{14}{16}\right) = -0.9739$$

$$f\left(-\frac{13}{16}\right) = -0.8434$$

$$f\left(-\frac{12}{16}\right) = -0.7254$$

$$f\left(-\frac{11}{16}\right) = -0.6193$$

$$f\left(-\frac{10}{16}\right) = -0.5243$$

$$f\left(-\frac{9}{16}\right) = -0.4398$$

$$f\left(-\frac{8}{16}\right) = -0.3651$$

$$f\left(-\frac{7}{16}\right) = -0.2996$$

$$f\left(-\frac{6}{16}\right) = -0.2426$$

$$f\left(-\frac{5}{16}\right) = -0.1935$$

$$f\left(-\frac{4}{16}\right) = -0.1515$$

$$f\left(-\frac{3}{16}\right) = -0.1163$$

$$f\left(-\frac{2}{16}\right) = -0.0870$$

$$f\left(-\frac{1}{16}\right) = -0.0632$$

$$f(0) = -0.0442$$

$$f\left(\frac{1}{16}\right) = -0.0295$$

$$f\left(\frac{2}{16}\right) = -0.0185$$

$$f\left(\frac{3}{16}\right) = -0.0107$$

$$f\left(\frac{4}{16}\right) = -0.0054$$

$$f\left(\frac{5}{16}\right) = -0.0023$$

$$f\left(\frac{6}{16}\right) = -6.7490e - 04$$

$$f\left(\frac{7}{16}\right) = -8.4049e - 05$$

$$f\left(\frac{8}{16}\right) = 0$$

$$f\left(\frac{9}{16}\right) = 8.3433e - 05$$

$$f\left(\frac{10}{16}\right) = 6.6504e - 04$$

$$f\left(\frac{11}{16}\right) = 0.0022$$

$$f\left(\frac{12}{16}\right) = 0.0053$$

$$f\left(\frac{13}{16}\right) = 0.0103$$

$$f\left(\frac{14}{16}\right) = 0.0177$$

$$f\left(\frac{15}{16}\right) = 0.0280$$

$$f\left(\frac{16}{16}\right) = 0.0417$$

$$M(1) = -0.0884$$

$$M(2) = -0.3651$$

$$M(4) = -0.4385$$

$$M(8) = -0.4572$$

$$M(16) = -0.4618$$

$$T(1) = -1.2340$$

$$T(2) = -0.6612$$

$$T(4) = -0.5132$$

$$T(8) = -0.4758$$

$$T(16) = -0.4639$$

$$S(1) = -0.4702$$

$$S(2) = -0.4638$$

$$S(4) = -0.4634$$

$$\begin{aligned} S(8) &= -0.4633854 \\ S(16) &= -0.46338379 \end{aligned}$$

$$\text{Maple} : -0.4633836$$