Weierstrass Institute for
Applied Analysis and Stochastics

# Second Workshop on Nonsmooth and Stochastic Optimization - NOSTOP 2018

**dedicated to the 70th birthday of** *Werner Römisch*

NOSTOP 2018
June 26th 2018

## Schedule

**Tuesday, June 26th 2018**

| Time | Title |
|---|---|
| 09:30 | Coffee and Registration |
| 10:00 | Welcome<br>*René Henrion* |
| 10:30 | N.N.<br>*Caren Tischendorf* |
| 11:00 | N.N.<br>*Michael Hintermüller* |
| 11:30 | Coffee break |
| 12:00 | Extended Euler-Lagrange and Hamiltonian Conditions in Optimal Control of Sweeping Processes with Controlled Moving Sets<br>*Boris Mordukhovich* |
| 12:30 | Characterizations of the free disposal condition for nonconvex economies on infinite-dimensional commodity spaces<br>*Abderrahim Jourani* |
| 13:00 | Lunch (Caffè Kamee) |
| 14:30 | Isolated calmness of Hölder and Lipschitz type in nonlinear optimization<br>*Diethard Klatte* |
| 15:00 | Optimal time delays in a class of reaction-diffusion equations<br>*Fredi Tröltzsch* |
| 15:30 | On Cournot-Nash-Walras equilibria and their computation<br>*Jiří Outrata* |
| 16:00 | Coffee break |
| 16:30 | Selected Problems in Function Spaces and their Numerical Treatment<br>*Christian Grossmann* |
| 17:00 | Approximation of stochastic processes<br>*Alois Pichler* |
| 17:30 | Power, Gas, and Werner Römisch<br>*Rüdiger Schultz* |
| 18:00 | Get-together (Humboldt-Kabinett) |

## *Hommage to Werner Römisch*

This workshop is dedicated to the 70th birthday of one of the pioneers of stochastic optimization. The mathematical work of Werner Römisch is closely tied with Humboldt University, where he graduated (1971), received his PhD (1976), defended his habilitation thesis (1984) and worked as a lecturer and full professor (from 1993 to 2013) at the Department of Mathematics.

Werner Römisch wrote approximately 150 papers and co-authored 3 monographcs. His early research was devoted to random differential equations and shifted then, around the mid-eighties, to stochastic optimization, i.e., optimization problems affected by random parameters. He strongly promoted this area by his fundamental contributions to the stability theory of stochastic optimization problems, to scenario reduction and generation methods and, more recently, to the investigation of Quasi Monte-Carlo methods in the context of stochastic programming. Apart from its theoretical impact, his work has been very much inspired by real life applications, first of all in power management. He supervised 12 PhD theses and beyond that significantly influenced the academic career of many more young researchers.

His research on stochastic optimization led Werner Römisch in a natural way to the need of applying tools from variational and set-valued analysis as well as from nonsmooth optimization. Consequently, there is no better patron to this second Berlin workshop NOSTOP 2018 on stochastic and nonsmooth optimization than him.

## Sponsors

This workshop benefits from the support of the FMJH *Program Gaspard Monge in optimization and operation research* (PGMO), and by the support of the Weierstrass Institute Berlin.

# On the Approximate Solution of Random Operator Equations

*Werner Römisch*

1981

## 1. Introduction

In the late fifties the Prague school of probabilists around Špaček and Hanš began to investigate random operator equations (cf. [19]), which resulted in a unified treatment of various stochastic models (see [1]).

Especially the contributions by A. T. Bharucha-Reid, his book [1] and the survey [2] initiated an essential improvement of the theory and the approximation of random operator equations some years ago. The works by Engl [15] and Nowak [28] satisfactorily explained the measurability of solutions of such equations on the basis of the fast development of the theory of measurable multifunctions and measurable selectors (see [21, 26, 33, 37]). But in the meantime there have been published several works on the approximation of random operator equations and their random solutions, too. Let us mention [3, 10, 16 and 27,

## 2. Measurability and random operators

Let $\boxed{(\Omega, \mathfrak{A}, P) \text{ be a complete probability space}}$ and $X$, $Y$ be real separable Banach spaces. By $\mathfrak{B}(X)$ we denote the $\sigma$-algebra of Borel sets of $X$, i.e. the one generated by the open sets of $X$, and by $\mathfrak{A} \otimes \mathfrak{B}(X)$ the smallest $\sigma$-algebra containing $\{AxB \mid A \in \mathfrak{A}, B \in \mathfrak{B}(X)\}$. Further we define $\mathfrak{P}(X) := \{D \mid D \subseteqq X, D \neq \emptyset\}$ and $\mathrm{Cl}\,(X) := \{D \mid D \in \mathfrak{P}(X), D \text{ closed}\}$. A map $C: \Omega \to \mathfrak{P}(X)$ is called a $\boxed{\text{multifunction}}$ and we define

$\mathrm{Gr}\, C := \{(\omega, x) \in \Omega x X \mid x \in C(\omega)\}$. $C: \Omega \to \mathfrak{P}(X)$ is called measurable if for all open $B \subseteqq X$ $\{\omega \in \Omega \mid C(\omega) \cap B \neq \emptyset\} \in \mathfrak{A}$ (weakly measurable in [21]). Let

$S(C) := \{x \mid x: \Omega \to X \text{ measurable}, x(\omega) \in C(\omega) \text{ for all } \omega \in \Omega\}$

be the set of all $\boxed{\text{measurable selectors}}$ of $C$ and we say that $C$ has a Castaing representation if there exist $x_m \in S(C)$, $m \in N$, 4.6 Theorem: Let $\alpha \in ]0,1[$ and $A \in \mathcal{L}(\mathbb{R}^m, \mathbb{R}^s)$ have rank $s \leq m$. Assume that the distribution function $F_\mu$ of $\boxed{\mu \in \mathcal{P}(\mathbb{R}^s)}$ is Lipschitzian and satisfies the following "(local) inverse Lipschitz condition at $\alpha$":

metric regularity

$$\left.\begin{array}{l} \text{There exist positive constants } \gamma,\ \delta_1 \text{ and } \delta_2 \text{ such that} \\ F_\mu(z+\bar{\varepsilon}) \geq F_\mu(z) + \gamma\varepsilon \\ \text{for all } \varepsilon \in [0,\delta_2],\ \bar{\varepsilon} := (\varepsilon,\dots,\varepsilon),\ z \in F_\mu^{-1}([\alpha-\delta_1, \alpha+\delta_1]) \end{array}\right\} \quad (4.8)$$

Then there is a constant $L_0 > 0$ such that

$$D(C_\alpha(\mu), C_\alpha(\nu)) \leq L_0 \rho(\mu,\nu)$$

for all $\nu \in \mathcal{P}(\mathbb{R}^s)$ with sufficiently small $\rho(\mu,\nu)$.

**Proposition 2.1.** Let the distribution function $F_\mu$ of $\mu \in \mathscr{P}(\mathbb{R}^s)$ be locally Lipschitzian, $p_0 \in (0,1)$; let $X_0$ be a closed set; and let $x_0 \in X_0$ be such that $F_\mu(Ax_0) \geq p_0$. In the case $F_\mu(Ax_0) = p_0$, let further $\boxed{\partial F_\mu(Ax_0) \cap N_{X_0}(x_0) = \emptyset,}$ where $\partial$ denotes the Clarke generalized gradient of $F_\mu(A\,\cdot\,)$ and $N_{X_0}(x_0)$ is the Clarke normal cone to $X_0$ at $x_0$ (see Ref. 35). Then, the multifunction $p \mapsto \{x \in X_0 : F_\mu(Ax) \geq p\}$ is $\boxed{\text{pseudo-Lipschitzian}}$ at $(x_0, p_0)$.

**Theorem 2.5.** *Consider the parametric program* $P(t)$, *fix some* $t^0 \in T$.

(i) *Assume that there exists a bounded open subset* $V$ *of* $\mathbb{R}^m$ *and a nonempty subset* $X$ *of* $V$ *such that* $X = \psi_V(t^0)$ (*i.e.* $X$ *is a bounded* $\boxed{\text{CLM set}}$ *for* $f(\cdot, t^0)$ *on* $M(t^0)$).

(ii) *Let the multifunction* $M$ *be closed-valued and closed at* $t^0$.

(iii) *Suppose* $M$ *is* $\boxed{\text{pseudo-Lipschitzian*}}$ *at each pair* $(x^0, t^0)$ *belonging to* $\psi_V(t^0) \times \{t^0\}$.

(iv) *Suppose there are real numbers* $p \in (0, 1]$, $L_f > 0$ *and* $\delta_f > 0$ *such that* $f(\cdot, t)$ *is continuous and*

$$|f(x, t^0) - f(y, t)| \leqslant L_f(\|x - y\| + d(t, t^0)^p)$$

*for each* $x, y \in \operatorname{cl} V$ *and each* $t \in T$ *satisfying* $d(t, t^0) < \delta_f$.

*Then we have*:

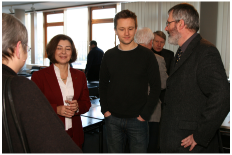
(a) *The multifunction* $\psi_V$ *is upper semicontinuous* (*u.s.c.*) *at* $t^0$, *i.e. for each* $\varepsilon > 0$ *there exists* $\eta > 0$ *such that*

$$\psi_V(t) \subset \psi_V(t^0) + \varepsilon B_m \quad \text{whenever } d(t, t^0) < \eta.$$

(b) *There exist positive reals* $\delta_\varphi$ *and* $L_\varphi$ *such that* $\psi_V(t)$ *is a CLM set for* $f(\cdot, t)$ *on* $M(t)$ *and* $|\varphi_V(t) - \varphi_V(t^0)| \leqslant L_\varphi d(t, t^0)^p$ *whenever* $d(t, t^0) < \delta_\varphi$. $\square$

## Werner Römisch and his PhD students

- Matthias Gelbrich (1990)
- Marta Lourdes Baguer (1990)
- Lulia E. Cuesta (1991)
- Nicole Gröwe (1995)
- Matthias Nowak (2000)
- Appolinaire Nzali (2001)
- Andreas Eichhorn (2007)
- Holger Heitsch (2007)
- Christian Küchler (2008)
- Thorsten Sickenberger (2008)
- Stefan Vigerske (2012)
- Hernan Leövey (2015)



Best player of table tennis in the stochastic programming community

## Nicest handwriting in the stochastic programming community

Introduction

General stochastic program (deterministic equivalent):

$P(\mu)$ $\quad \min \left\{ \int_{\mathbb{R}^s} f(x,\xi)\mu(d\xi) : x \in M(\mu) \right\}$

where $M(\mu) := \left\{ x \in X : \int_{\mathbb{R}^s} f_j(x,\xi)\mu(d\xi) \leq 0,\ j = 1,\dots,d \right\}$

$X \subset \mathbb{R}^m$ is (nonempty) closed (compact),

$f, f_j : \mathbb{R}^m \times \mathbb{R}^s \longrightarrow \overline{\mathbb{R}} = \mathbb{R} \cup \{+\infty\}$ are normal integrands $(j = 1,\dots,d)$,

$\mu$ is a (Borel) probability measure on $\mathbb{R}^s$, $\mu \in \mathcal{P}(\mathbb{R}^s)$.

$v(\mu) := \inf \left\{ \int_{\mathbb{R}^s} f(x,\xi)\mu(d\xi) : x \in M(\mu) \right\}$ (minimum)

$S_\varepsilon(\mu) := \varepsilon\text{-}\operatorname{argmin} \left\{ \int_{\mathbb{R}^s} f(x,\xi)\mu(d\xi) : x \in M(\mu) \right\}$ $(\varepsilon \geq 0)$

$= \left\{ x \in M(\mu) : \int_{\mathbb{R}^s} f(x,\xi)\mu(d\xi) \leq v(\mu) + \varepsilon \right\}$

(set of $\varepsilon$-minimizers)

$S(\mu) := \operatorname{argmin} \left\{ \int_{\mathbb{R}^s} f(x,\xi)\mu(d\xi) : x \in M(\mu) \right\} = S_0(\mu)$

(set of minimizers)

Problem:

(Quantitative) continuity properties of $v$, $S_\varepsilon$, $S$ with respect to perturbations of $\mu$!

Perturbations of $\mu$ are measured in terms of convergences and metric distances of probability measures.

Best grandfather in the stochastic programming community

Biggest gourmet in the stochastic programming community

## However: when the enigne goes on strike

## However: when the enigne goes on strike

These operators are so random, I can‘t fix them in my mind

QMC can be so boring!

# What is the cooperation number between Ute and Werner Römisch?

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**Theorem**

*The cooperation number between Ute and Werner Römisch is at most 5!*

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**Conjecture**

*The cooperation number equals 5.*

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**Theorem**

*The cooperation number between Ute and Werner Römisch is at most 5!*

**Conjecture**

*The cooperation number equals 5.*

Theorem correct, conjecture false.

## Ute Römisch

## Werner Römisch

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## Werner Römisch

## The prosaic way

If I knew about the existence of a cooperation number calculator (MathSciNet) I could have saved a lot of efforts to see that it equals 4 in our case:

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Werner Römisch $\Longrightarrow$ S. Rachev $\Longrightarrow$ A. Obretenov $\Longrightarrow$ D. Vandev $\Longrightarrow$ U. Römisch

## Ad multos annos!