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Lattice Boltzmann Methods for Fluid-Structure Interaction Problems

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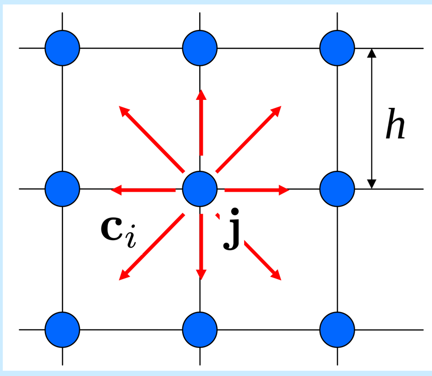
INTRODUCTION

We investigate extensions of the lattice Boltzmann method towards fluid-structure interaction problems.
Focusing on a D2Q9 model, we consider a LBGK evolution

$$(1) \quad f_i(n+1, \mathbf{j}+\mathbf{c}_i) = f_i(n,\mathbf{j}) + \frac{1}{\tau}\left(f_i^{eq}(f) - f_i\right)(n,\mathbf{j})$$



on a regular *h*-spaced lattice, with equilibrium function

$$f_i^{eq} = f_i^* \rho + f_i^* c_s^{-2} \mathbf{c}_i \cdot \mathbf{u} + f_i^* \frac{c_s^{-4}}{2}\left((\mathbf{c}_i \cdot \mathbf{u})^2 - c_s^2 \mathbf{u}^2\right)$$

where f_i^*, c_s are weights depending on the particular LBM realization.

Asymptotic Analysis

The numerical solution of (1) can be predicted using an asymptotic expansion:

$$(2) \; F_i = f_i^{(0)} + h f_i^{(1)} + h^2 f_i^{(2)}$$

$$(3) \quad f^{(0)} = f_i^*, \quad f^{(1)} = c_s^{-2} f_i^* \mathbf{c}_i \cdot \mathbf{u}, \quad f^{(2)} = c_s^{-2} f_i^* \left(p - \frac{c_s^{-2}}{2}\left((\mathbf{c}_i \cdot \mathbf{u})^2 - c_s^2 \mathbf{u}^2\right) - \tau \mathbf{c}_i \cdot \nabla \mathbf{u} \cdot \mathbf{c}_i\right)$$

whose coefficients can be defined [5] inserting the ansatz (2) into (1), as functions of pressure and velocity (solution of Navier-Stokes equations).

From (3), the prediction can be written as a sum of *equilibrium+ non-equilibrium*, the latter depending on velocity gradients

$$(4) \; F_i = f_i^{eq}\left(1 + h^2 c_s^{-2} p, h\mathbf{u}\right) + f_i^{neq,(2)}(\nabla \mathbf{u})$$

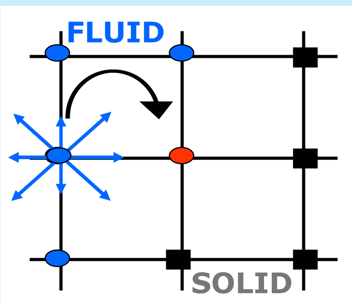
MOVING BOUNDARY LBM

An additional rule (**refill**) is needed to initialize the nodes entering the fluid domain (fig.1).

The **EQ+NE refill** initializes new fluid nodes approximating the interior prediction (4):

$$f_i(\text{new node}) = f_i^{eq}\left(1 + h^2 c_s^{-2} \tilde{p}, h\tilde{\mathbf{u}}\right) + \tilde{f}_i^{neq,(2)}$$

extrapolating *equilibrium* (pressure and velocity) and *non-equilibrium* part.



According to (4), a low order approximation of non-equilibrium is sufficient:
it can be copied from a neighbor

Note: using only the equilibrium part does not produce consistent results [2,4] (fig. 5b).

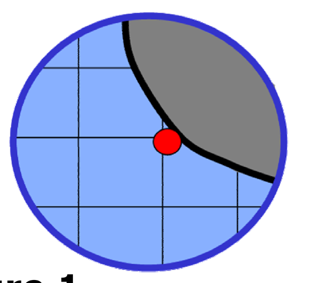


Figure 1.
Node initialization in moving boundary problems.

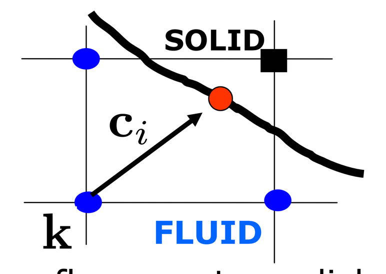
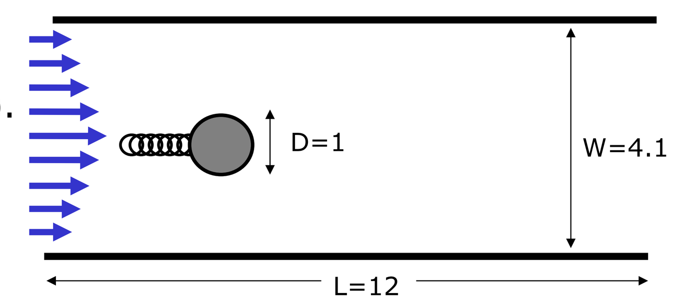


Figure 2. MEA approximates the force using the difference between outgoing and incoming momentum at boundary links.

Figure 3. Benchmark problem: flow past a disk, driven by a parabolic inlet profile. Geometry as in [8]: channel size length=12 x width=4.1, diameter D=1, center initial position x_C=(2,2). Reynolds = 50 Outflow: homogeneous Neumann BC



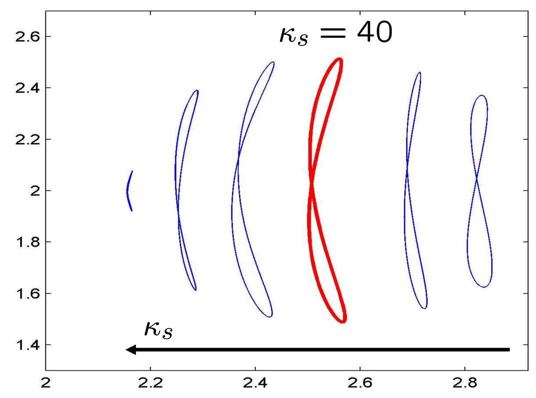
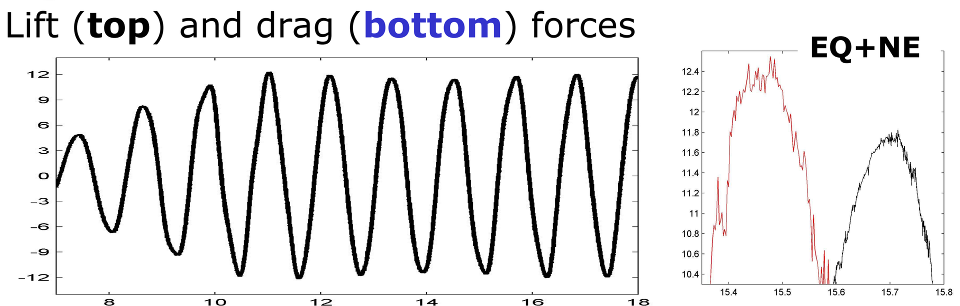


Figure 4.
Periodic trajectories of the disk, for different spring stiffness (from right to left):
$\kappa_s = 20, 25, 40, 60, 80, 100$
Grid size h=0.025.

Figure 5. $\kappa_s = 40$ (maximum oscillations):
Lift (**top**) and drag (**bottom**) forces



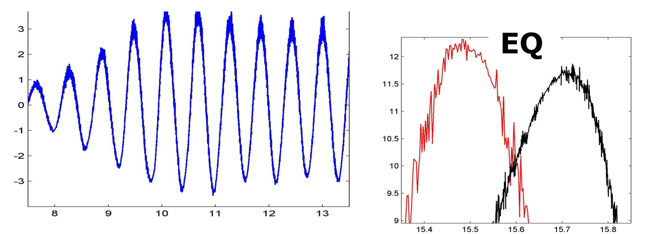
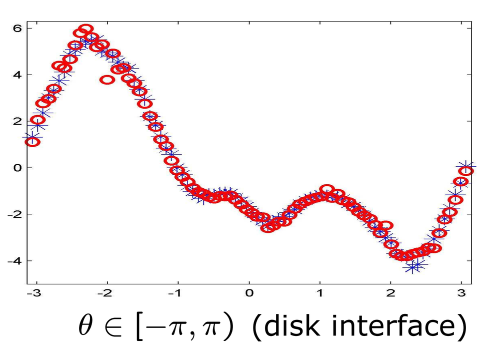
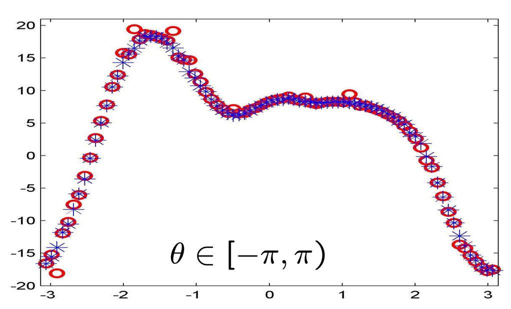


Figure 5b.
Irregular oscillations are first order in h using EQ+NE refill. Grids: h=**0.05**, **0.025**

Figure 6. Local stresses: horizontal (right) and vertical (below) forces along the interface.
ME-based (**red**) and pop-based (**blue**) extrapolations are compared.





BENCHMARK, RESULTS & DISCUSSION

We consider a channel flow past a moving disk, whose motion is constrained by a spring (fig.3).
Forces on the obstacle are computed with (6), and Newton equations are integrated (explicit Euler) for velocity and position of the disk.
Dirichlet BC on the disk surface implemented as in [1].

Fig. 4 shows the periodic trajectories of the center of the disk.

For the case of maximum oscillation amplitude, drag and lift forces on the disk are shown in fig. 5. Moving boundaries introduce grid oscillations. However, irregularities in the force decrease (first order in h) using a finer grid. This validates the accuracy of the EQ+NE refill algorithm, against an approximation only based on the equilibrium part (fig. 5b).

At a time when vertical force is maximum, we approximate the local interface stresses comparing *ME-based* and *pop-based* extrapolations (fig. 6).

Both methods provide similar results. Due to the moving boundary, irregularities may appear.

FORCE EVALUATION

Within LBM, forces on a solid obstacle can be efficiently computed using the *Momentum Exchange Algorithm* (MEA) [6] (fig.2).

At each boundary node, the momentum exchanged along each LB-link is computed (using the post-collision distribution pointing into the solid):

$$(5) \; \phi_i^{\text{MEA}}(\mathbf{k}) = f_i^C(\mathbf{k})\mathbf{c}_i - f_{i^*}(\mathbf{k})\mathbf{c}_{i^*}$$

the sum of the contributions (5) along the boundary is used to approximate hydrodynamic force.

Corrected Momentum Exchange

Using (3), we found that the following correction is needed to obtain a Galilean invariant (in relevant orders) force computation:

$$(6) \quad \phi_i^{\text{CME}} = \phi_i^{\text{MEA}} - c_s^{-4}\left((\mathbf{c}_i \cdot \mathbf{u}_B)^2 - c_s^2 \mathbf{u}_B^2\right)\mathbf{c}_i$$

(where $\mathbf{u}_B$ is the velocity at the boundary).

The following asymptotic expansion for (6) holds:

$$(7) \; \phi_i^{\text{CME}} = h^2 2 c_s^{-2} f_i^* \left(p + c_s^{-2} \nu \mathbf{c}_i \cdot \nabla \mathbf{u} \cdot \mathbf{c}_i\right) + O(h^3)$$

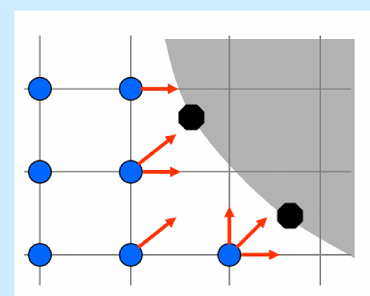
Accuracy Results [2,3]

- Corrected ME provides an accurate (first order in h) global force evaluation
- CME is consistent also for *Lees-Edwards* BC (periodic in Galilean-transformed systems), useful tool in suspension simulations [7]
- Local interface stresses are approximated only up to order $O(\sqrt{h})$

Local Boundary Forces

At selected points of the interfaces (●), two extrapolation methods are investigated (fig.6):

(A) **ME-based extrapolation**: stresses are extrapolated from (6), using the expansion of the momentum exchanged (7)



(B) **pop-based extrapolation**:
LB distributions are approximated on the boundary, extracting then the stresses using (3)

(A) is more efficient within the LBM (no need of off-grid extrapolation), while (B) might be better in terms of stability, since it can be combined with different extrapolation rules.

Outlook

The results show how asymptotic analysis can be used to understand the LBM and to extend it to fluid-structure interaction problems, efficiently (limiting the additional computational effort) and consistently (without spoiling the accuracy of the standard scheme).

Moving Boundary LBM and evaluation of fluid-solid forces have been investigated. The described approaches offer wide space for further improvement. Applications to dense suspension flow and deformable interfaces are object of current research.

Acknowledgments: A. Caiazzo is thankful to Michael Junk for his supervision.
This research has been funded by the DFG (*Deutsche Forschungsgemeinschaft*) and it is currently supported by the European Commission, through the COAST project (EU-FP6-IST-FET Contract 033664).

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