

# Adaptive method for the digitization of mathematical journals

IMU-WDML Workshop

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InftyProject ((<http://www.inftyproject.org>))

Science Accessibility Net (<http://www.sciaccess.net>)

# Plan of the talk

- About InftyProject
- Making Rich Digital Mathematical Libraries
  - Process Flow and Technical Components
- Current State of the Art with Demonstration
- Adaptive Method
  - Character and Symbol Recognition
  - Logical Structure Analysis
- Future Problems

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# *Section 1*

## About Infty Project

# InftyProject

- R&D on *Math Information Systems*

- Main system development

  - InftyReader : Math OCR software

  - InftyEditor : Editor of math documents

    - Data conversion (XML, LaTeX, MathML, PDF, etc.)

  - ChattyInfty : InftyEditor + speech output, Authoring of DAISY

- URL :

  - Project site: <http://www.inftyproject.org/en/>

  - Release & user support of Infty products:

    - Science Accessibility Net <http://www.sciaccess.net/>

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### ■ Demonstration.

Recognition result samples (YMJ, AJM).

PageImages.iml - InftyEditor

File Edit Find Format TeX Function Setting Help

COROLLARY 6.  $(r(U_\varphi))^2 = \max\{|\lambda| : \det(U_{|\varphi|^2}|H-\lambda) = 0\}$ .

THEOREM 7.  $(r(U_\varphi))^2 = \lim_{n \rightarrow \infty} \left( \int_0^{2\pi} \prod_{l=0}^n |\varphi(k^l \theta)|^2 \frac{d\theta}{2\pi} \right)^{1/n}$

Proof. From the above lemma

$$b_0^{(n)} \leq \|\psi_n\|_\infty = \|U_{|\varphi|^2}^n\| \leq \sqrt{4N+1} \|\psi_n\|_2 = \sqrt{4N+1} \left( \sum_{l=-2N}^{2N} |b_l^{(n)}|^2 \right)^{1/2} \quad (1)$$

As  $|b_l^{(n)}| = |\alpha_{l0}^{(n)}| \leq b_0^{(n)}$ , therefore  $\sum_{l=-2N}^{2N} |b_l^{(n)}|^2 \leq b_0^{(n)2} (4N+1)$ . Hence

$$\sqrt{4N+1} \left( \sum_{l=-2N}^{2N} |b_l^{(n)}|^2 \right)^{1/2} \leq (4N+1) b_0^{(n)} \quad (2)$$

U sin  $b_0^{(n)2} \leq$  and  $(2) \|U_{|\varphi|^2}^n\| \leq (4N+1) b_0^{(n)}$ . Let  $4N+1=K$ . On taking limit as  $n \rightarrow \infty$  we obtain that

$$(r(U_\varphi))^2 = \lim_{n \rightarrow \infty} \left( \int_0^{2\pi} \prod_{l=0}^{n-1} |\varphi(k^l \theta)|^2 \frac{d\theta}{2\pi} \right)^{1/n}$$

SheetView - [YMJ\_51\_N1\_004.tif] [Zoom=20.00%]

File View Zoom Window

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PageImages.iml - InftyEditor

File Edit Find Format TeX Function Setting Help

FINITE LOCAL MONODROMY OF UNIT-ROOT F-ISOCRYSTALS

1181

Define

$$Y = \lim_{n \rightarrow \infty} Y_N Y_{N+1} \cdots Y_{n-1} Y_n$$

Then the right-hand side of the above equation is convergent in  $GL_r(O_E)$  and satisfies the relation

$$A\sigma(Y) = Y$$

Since  $M$  is étale,  $Y$  is contained in  $GL_r(O_E^\dagger)$  by Proposition 3.1.5. Therefore, the assertion of Proposition 5.2.1 follows Lemma 3.3.2.

6. Proof of Lemma 5.2.4. In this section we prove Lemma 5.2.4 using  $p$ -adic analysis. Assume that the residue class field  $k$  of  $F$  is algebraically closed throughout this section.

(6.1) Put  $m \geq \alpha = (x \in O_{\hat{k}alg} \mid |x| \leq |p|^\alpha)$  for any positive number  $\alpha$ . Let  $O_{\hat{k}alg}[[z]]$  be the ring of formal power series and denote by  $\phi$  the endomorphism on  $O_{\hat{k}alg}[[z]]$  which is defined by the identity on  $O_{\hat{k}alg}$  and by  $\phi(z) = z^p$ . We sometimes use the same notation  $m > \alpha$  for the ideal  $m > \alpha O_{\hat{k}alg}[[z]]$  in  $O_{\hat{k}alg}[[z]]$ .

$$Y = \lim_{n \rightarrow \infty} Y_N Y_{N+1} \cdots Y_{n-1} Y_n$$

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File View Zoom Window

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# *Section 2*

## Toward Rich DML

# Different levels in digitization


- Level 1: Bitmap images of printed materials  
e.g. GIF, TIFF
- Level 2: Searchable digitized document  
e.g. PDF with hidden text, Bib Link
- Level 3: Structured accessible document  
e.g. XML, HTML(+MathML), LATEX, ...
- Level 4: (partially) Executable document  
e.g. Mathematica, Maple
- Level 5: Formally presented document  
e.g. Mizar, OMDoc



# Different levels in digitization

- Level 1: **WDML achieved this level.**  
e.g. GIF, TIF
- Level 2: Searchable digitized document  
e.g. PDF with hidden text, Bib Link
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# Process Flow of Digitization

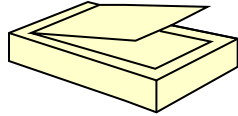


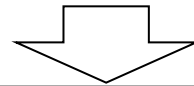
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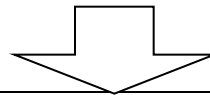
PDF

Texts &  
Math  
symbols

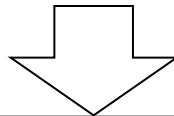
Layout Analysis : Segmentation of Areas (Text, Table, Figure)



Recognition per line  
(Character recognition, Math/Text segmentation, Math. Structure analysis)

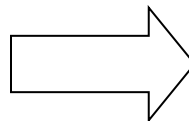


Document Structure analysis  
(Chapter, Section, Itemize, Theorem description, References, etc.)



XML

Outputs



LaTeX, XHTML+MathML,  
PDF, Braille codes, etc.

# Layout Analysis

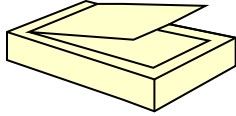
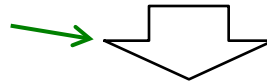


Image File (TIF)



PDF

(Pre processing)



Segmentation of Areas (Text, Table, Figure)

Sec. 10.4] SEQUENCES 595

**Example 4.**

Sequence	Limit points at:	Convergent or divergent
$1, 2, 3, \dots$	(none)	divergent
$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$	1	convergent
$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$	0	divergent
$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$	0 and 1	divergent

A number which appears infinitely often in a sequence is to be regarded as a limit point; this is a matter of convenience and convention.  
A sequence  $z_1, z_2, \dots$  is said to be **bounded**, if there is a positive number

Fig. 292. Last sequence in Example 4.

$K$  such that all the terms of the sequence lie in a disk of radius  $K$  about the origin, that is,

$$|z_n| < K \quad \text{for all } n.$$

For example, the second and the last sequences in Ex. 4 are bounded while the first and third are not. We observe that the two bounded sequences have limit points. This illustrates the following important theorem.

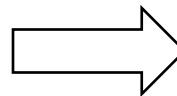
**Theorem 2 (Bolzano<sup>4</sup> and Weierstrass<sup>5</sup>).** A bounded infinite sequence has at least one limit point.

*Proof.* It is obvious that both conditions are necessary: a finite sequence cannot have a limit point, and the sequence  $1, 2, 3, \dots$ , though infinite, has no limit point because it is not bounded. To prove the theorem, consider a bounded infinite sequence  $z_1, z_2, \dots$  and let  $K$  be such that  $|z_n| < K$  for all  $n$ . If only finitely many values of the  $z_n$  are different, then, since the sequence is infinite, some number  $z$  must occur infinitely many times in the sequence, and, by definition, this number is a limit point of the sequence.

We may now turn to the case when the sequence contains infinitely many different terms. We draw the large square  $Q_0$  in Fig. 293 which contains all  $z_n$ . We subdivide  $Q_0$  into four congruent squares. Clearly, at least one of these squares (each taken with its

Fig. 293. Proof of Theorem 2.

<sup>4</sup> BERNHARD BOLZANO (1781–1848), German mathematician, a pioneer in the study of point sets.  
<sup>5</sup> Cf. footnote 3 in Sec. 10.3.



Sec. 10.4] SEQUENCES 595

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# Layout Analysis

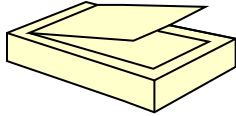
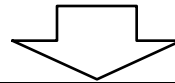


Image File (TIF)



PDF



Segmentation of Areas  $\Rightarrow$  Table Analysis

Sec. 10.4]

SEQUENCES

595

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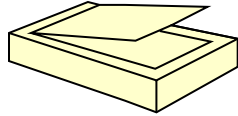


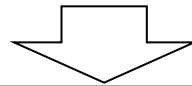
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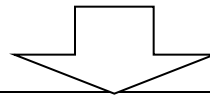
PDF

Texts &  
Math  
symbols

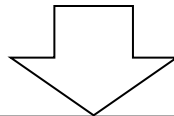
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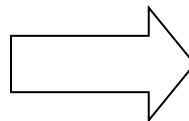


Document Structure analysis  
(Chapter, Section, Itemize, Theorem description, References, etc.)



XML

**Outputs**



LaTeX, XHTML+MathML,  
PDF, Braille codes, etc.

# Process Flow of Digitization

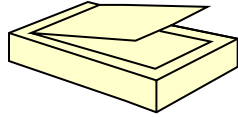


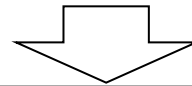
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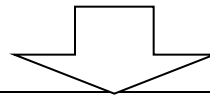
PDF

Texts &  
Math  
symbols

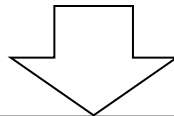
Layout Analysis : Segmentation of Areas (Text, Table, Figure)



Recognition per line  
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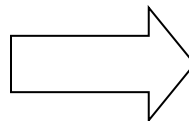


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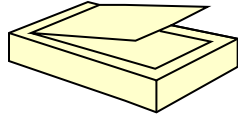


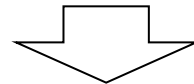
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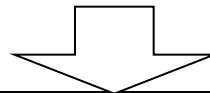
PDF

Texts &  
Math  
symbols

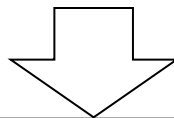
Layout Analysis : Segmentation of Areas (Text, Table, Figure)



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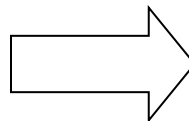


**Document Structure analysis**  
(Chapter, Section, Itemize, Theorem description, References, etc.)



XML

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LaTeX, XHTML+MathML,  
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# Document Structure Analysis

## ■ Detection of :

Title, Autor, Section, Subsection, Itemization, BibItem, Theorem, Lemma, etc.



- Currently, naïve methods are used:

Line classification using the combination features such as:

*Character size, Font Information (Bold, Italic, Small Capital), Keywords, Indentation, Starting with Numbers or Special pattern (e.g. “[Num]”), etc.*

- Stronger method is required in actual digitization.

## ■ Hyperlink inside document.

# *Section 3*

Current state of the art  
with demonstration

# “*InftyReader*” OCR software for math documents

- Demonstration...
  - Math recognition (*Already shown*)
  - Multi lingual recognition ← *FineReader OCR plug-in*  
*Czech paper result sample*
  - Matrices
  - Layout analysis, Table recognition
  - Logical structure analysis

# “InftyReader” OCR software for math documents

## ■ Demonstration

### ■ Math recognition

### ■ Multi language

Czech paper

Matrices

### ■ Layout analysis

### ■ Logical structure

Czech\_Algebra.iml - InftyEditor

File Edit Find Format TeX Function Setting Help

Utvořme množinu  $\Pi \times \Phi$  všech dvojic  $(\pi, \varphi)$ , kde  $\pi \in \Pi$ ,  $\varphi \in \Phi$ . Definujme  
na množině  $\Pi \times \Phi$  operaci takto:  $\downarrow$

$$(\pi_1, \varphi_1) \cdot (\pi_2, \varphi_2) = (\pi, \varphi), \downarrow$$

kde prvky  $\pi, \varphi$  jsou určeny vztahy  $\downarrow$

$$\pi(m) = \pi_1(\pi_2(m)), \downarrow$$

$$\varphi(m) = \varphi_1(\pi_2(m)) \cdot \varphi_2(m)^* \quad (\alpha) \downarrow$$

Pak je  $\Pi \times \Phi$  grupa.  $\downarrow$

Důkaz. I.  $\downarrow$

$$[(\pi_1, \varphi_1) \cdot (\pi_2, \varphi_2)] \cdot (\pi_a, \varphi_a) = (\pi, \varphi) \cdot (\pi_3, \varphi_3) = (\pi', \varphi'), \downarrow$$

kde pro  $\pi, \varphi$  platí  $(\alpha)$  a pro  $\pi', \varphi'$  platí  $\downarrow$

$$\pi'(m) = \pi_1(\pi_2(\pi_a(m))), \downarrow$$

$$\varphi'(m) = \varphi(\pi_a(m)) \cdot \varphi_3(m) = \varphi_1(\pi_2(\pi_3(m))) \cdot \varphi_2(\pi_3(m)) \cdot \varphi_3(m) \downarrow$$

SheetView - [0030.tif] [Zoom=20.00%]

File View Zoom Window

Utvořme množinu  $\Pi \times \Phi$  všech dvojic  $(\pi, \varphi)$ , kde  $\pi \in \Pi$ ,  $\varphi \in \Phi$ . Definujme  
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Pak je  $\Pi \times \Phi$  grupa.

Důkaz. I.

# “InftyReader” OCR software for math documents

## ■ Demonstrates

■ Math recognition

■ Multi language

Czech paper

■ Matrices ←

■ Layout analysis

■ Logical structure

The screenshot shows two windows from the InftyEditor software. The top window, titled "Matrices.iml - InftyEditor", displays the original scanned text from a Czech document. It contains a matrix  $A \equiv (a_1 a_2 \dots a_n) \equiv$  with elements  $a_{i,j}$  and vectors  $x \equiv (x_1 x_2 \dots x_n)^t$ ,  $l \equiv (l_1 l_2 \dots l_n)^t$ , and  $u \equiv (u_1 u_2 \dots u_n)^t$ . The bottom window, titled "SheetView - [010.tif] [Zoom=20.00%]", shows the OCR-processed version of the same content, where the mathematical symbols are rendered in a clean, digital font.

# “InftyReader” OCR software for math documents

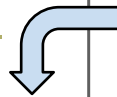
## Demonstration

- Math recognition

- Multi language

*Czech paper*

Matrices



- Layout analysis

- Logical structure

AIF\_1999\_639.iml - InftyEditor

File Edit Find Format TeX Function Setting Help

LUBOMIR GAVRILOV  
639  
Proof - The critical values of  $H$  are easily computed (Table 1).

$a < -\frac{1}{2}$	$h_1^c < h_{34}^s < h_2^c$
$a = -\frac{1}{2}$	$h_1^c < h_{34} = h_2$
$-\frac{1}{2} < a < -\frac{1}{3}$	$h_1^c < h_{34} < h_2^s$
$a = -\frac{1}{3}$	$h_1^c - h_{34} < h_2^s$
$-\frac{1}{3} < a < 0$	$h_{34} < h_1^c < h_2^s$
$a = 0$	$h_1^c < h_2^s$
$0 < a < 1$	$h_1^c < h_2^s < h_{34}^s$
$a > 1$	$h_1^c < h_{34}^s < h_2^s$
$a = \infty$	$h_1^c < h_{34}^s$

SheetView - [AIF\_1999\_639.tif] [Zoom=30X]

File View Zoom Window

Proof. The critical values of  $H$  are easily computed (Table 1).

$a < -\frac{1}{2}$	$h_1^c < h_{3,4}^s < h_2^c$
$a = -\frac{1}{2}$	$h_1^c < h_{3,4} = h_2$
$-\frac{1}{2} < a < -\frac{1}{3}$	$h_1^c < h_{3,4} < h_2^s$
$a = -\frac{1}{3}$	$h_1^c = h_{3,4} < h_2^s$

# “InftyReader” OCR software for math documents

## ■ Demonstration

■ Math reader

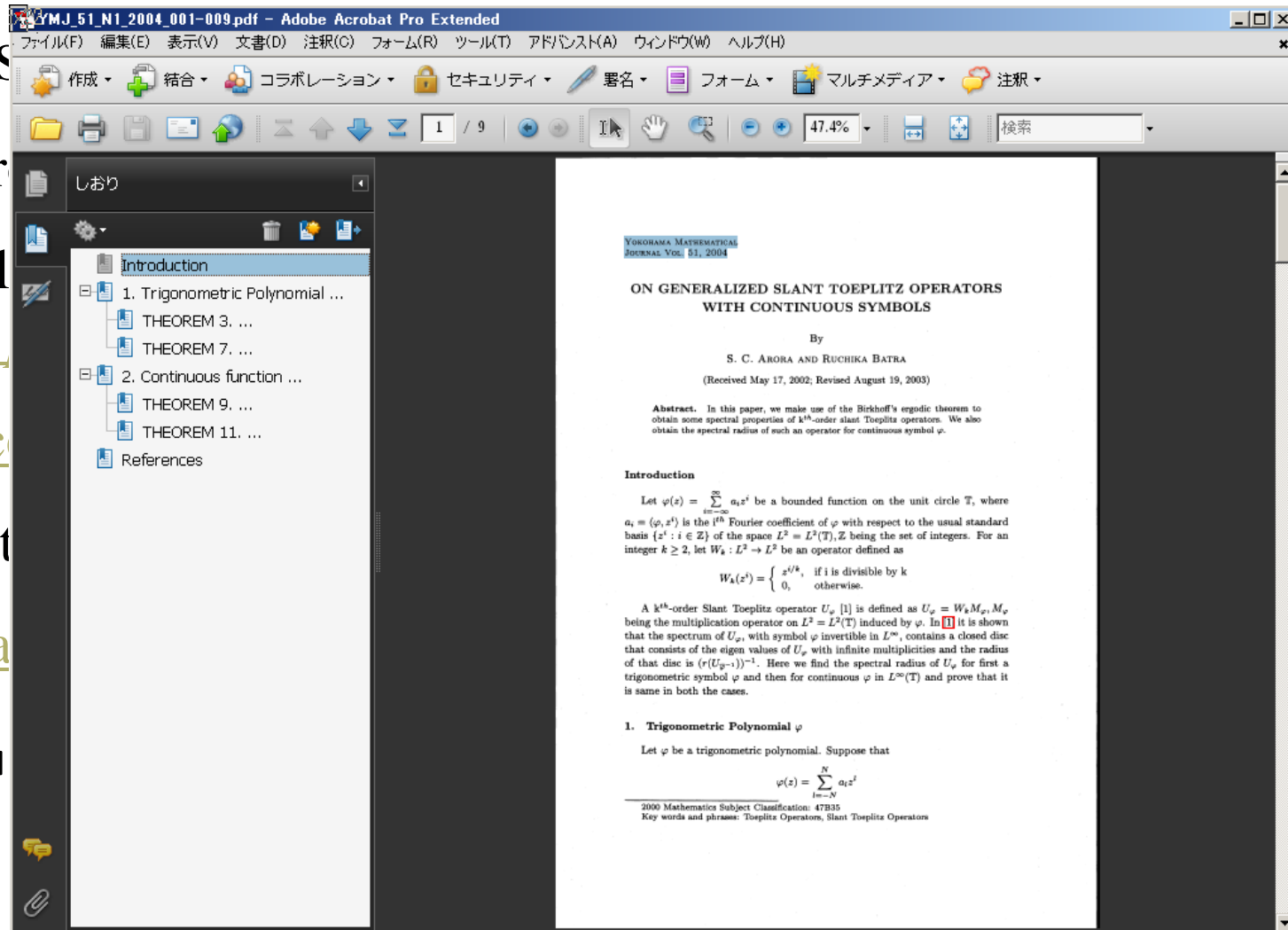
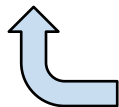
■ Multiple

Czech

■ Matrix

■ Layout

■ Logical



# “*InftyReader*” OCR software for math documents

- Demonstration...
  - Math recognition (*Already shown*)
  - Multi lingual recognition ← *FineReader OCR plug-in*  
*Czech paper result sample*
  - Matrices
  - Layout analysis, Table recognition
  - Logical structure analysis



# *Section 4*

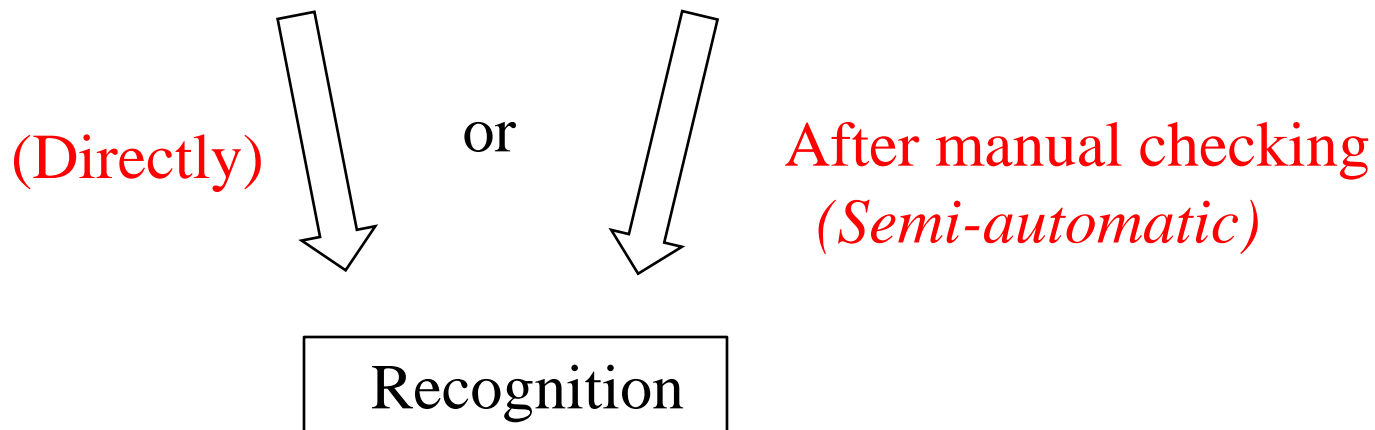
## Large Volume Recognition

# Large Volume Digitization

## ■ *Adaptive method* is efficient:

Get information **from the target document**:

- *Character features,*
- *Math formula parameters,*
- *Layout parameters, etc.*



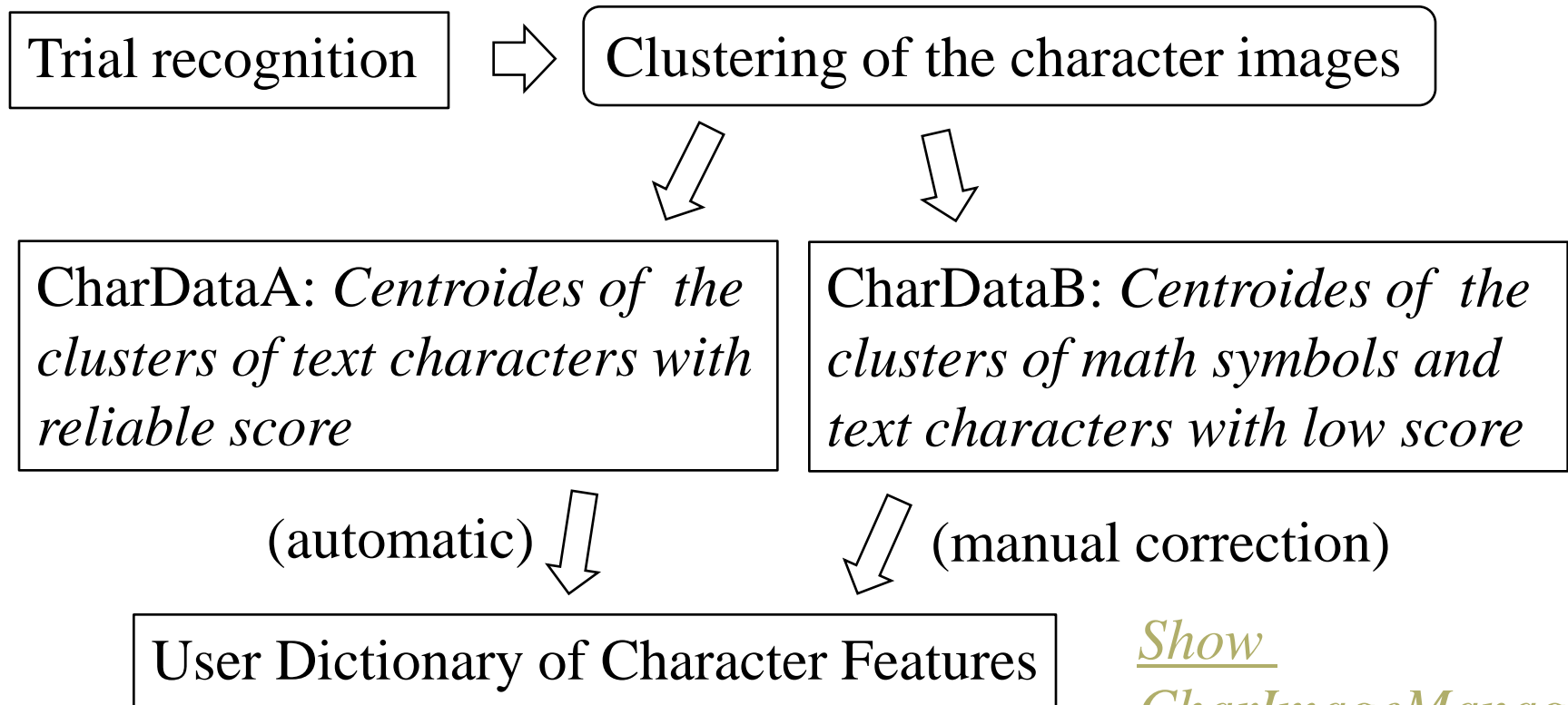
# Large Volume Digitization

## ■ Process Flow using *BatchInfty* & *InftyReader pro*

1. Noise reduction, centering, etc.
2. Trial recognition
3. Extraction features:
  - Document style → Logical structure analysis
  - **Character cluster images** → **OCR engine**
4. Recognition & verification
5. PDF output

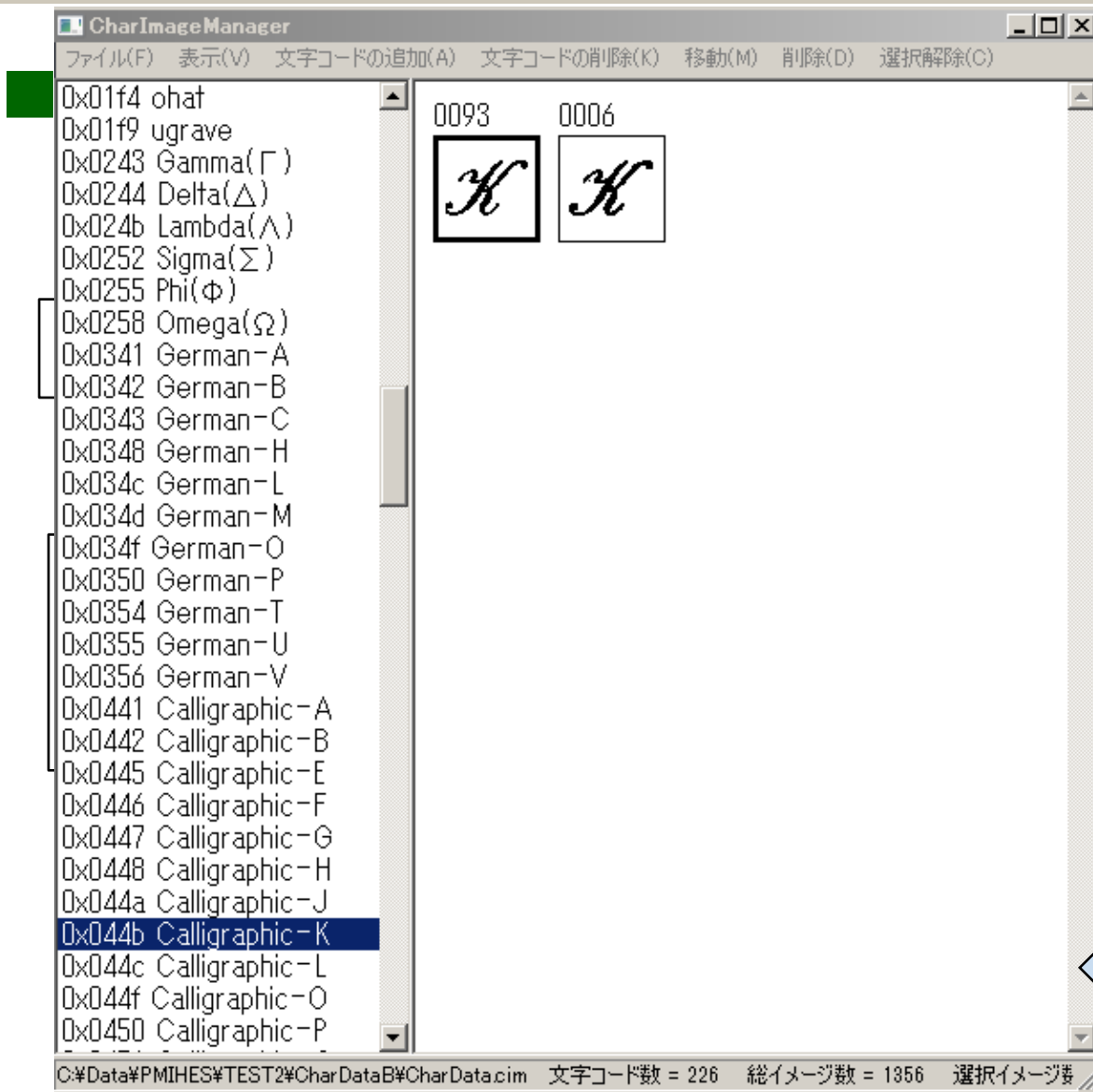
# Large Volume Digitization

- Generation of *UserDictionary* adapting OCR engine to the target documents.



Show  
CharImageManager

# Large Volume Digitization



Adapting OCR

Character images

B: Centroides of the  
of math symbols and  
acters with low score

ual correction)

← Show  
CharImageManager

---

# *Section 5*

## Open Problems

# Problems

- Further improvement of character/symbol recognition and structure analysis of math expressions.
  - Touched characters, Broken characters in math area
  - Low resolution image
  - Different type face (Old books, typewriter prints, etc.)
  - Bold char detection in math area

# Problems

- Logical Structure Analysis (Automatic detection and manual correction) --- *still difficult!*
  - Title, Autor, Section, Subsection, Itemization, BibItem, Theorem, Lemma, etc.
  - Hyperlink inside document.



# Problems

- Detection/Analysis of Figures and Tables
  - Detection of characters in figures
  - Table structure analysis (Sample)
  - Diagram recognition
    - Chemical diagrams ← *Recently developing world wide*
    - (Commutative diagrams) ← *Future work*

# Problems

## ■ Detection/Analysis of Figures and Tables

- Detection of characters in figures
- Table structure analysis (*Sample*)
- Diagram recognition



- Chen
- (*Com*)

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Example 4.

Sequence	Limit points at:	Convergent or divergent
$1, 2, 3, \dots$	(none)	divergent
$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$	1	convergent
$\frac{1}{2}, 2, \frac{1}{3}, 3, \frac{1}{4}, 4, \dots$	0	divergent
$\frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{4}{5}, \frac{1}{6}, \frac{5}{6}, \dots$	0 and 1	divergent

A number which appears infinitely often in a sequence is to be regarded as a limit point; this is a matter of convenience and convention.

A sequence  $z_1, z_2, \dots$  is said to be **bounded**, if there is a positive number

# Problems

## ■ Detection/Analysis of Fibrations

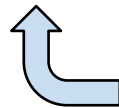
■ Detection of characters in fibrations

■ Table structure analysis (*Sam*)

■ Diagram recognition

● Chemical diagrams ← *K*

● (Commutative diagrams)



where

$$\tilde{M} = \tilde{\psi}^*(2K_P + R_P) - \sum \mathcal{E}_i.$$

The bicanonical map  $\Phi_{2K}$  of  $S$  can be decomposed as follows ([4])

$$\Phi_{2K} : S \xrightarrow{\hat{\Phi} \circ \rho^{-1}} \hat{P} \xrightarrow{\mu} \mathbf{P}^{p_2(S)-1}$$

where  $\mu$  is defined by the linear system  $|\tilde{M}|$ .

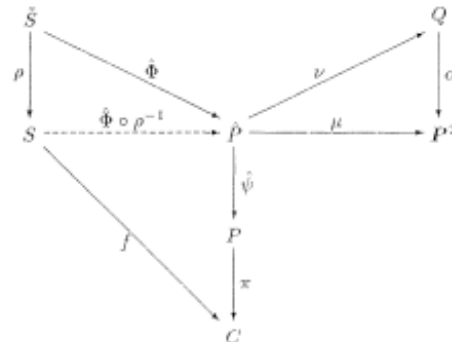
A theorem of Xiao[4, Théorème 5.5] claims that if  $S$  is a minimal surface of general type with  $p_2(S) \geq 3$  and which has a pencil of genus 2, then  $|2K_S|$  has neither fixed part nor base point. In such case,  $\mu$  is a morphism.

**Proposition 2.1** *Let  $S$  be a regular minimal surface of general type with a pencil of genus 2. If  $2 \leq K_S^2 = p_g(S) \leq 3$  or  $1 \leq K_S^2 < p_g(S) \leq 3$  (i.e. the cases (a)-(d) in Section 1), then there is only one hyperelliptic involution.*

*Proof.* If  $S$  has another hyperelliptic involution  $\sigma_2$ , then there is a ruled surface  $\tilde{P}_2$  and a morphism  $\mu_2 : \tilde{P}_2 \rightarrow \mathbf{P}^{p_2(S)-1}$  such that  $\Phi_{2K} = \mu_2 \circ \hat{\Phi}_2 \circ \rho^{-1}$ . Since  $\tilde{P}_2 \neq \tilde{P}$ , we should have  $\deg \Phi_{2K} > 2$ . But by a theorem of Xiao[4, Théorème 5.6], we have  $\deg \Phi_{2K} = 2$  because  $p_2(S) \geq 4$  in this case. This is a contradiction.  $\square$

**Proposition 2.2** *Let  $S$  be a regular minimal surface of general type with  $K_S^2 = p_g = 1$  (i.e. the case (e) in Section 1). Then there are at most 2 hyperelliptic involutions.*

*Proof.* Suppose that  $S$  has a hyperelliptic involution  $\sigma$  induced by a genus 2 fibration  $f : S \rightarrow C$ . In this case we have  $e = 1$ ,  $n = 6$ ,  $s_3(f) = 3$ . Hence  $\tilde{\psi}$  is composed of 6 blow-ups. Since  $p_2(S) = 3$ ,  $\deg \Phi_{2K} = 4$ ,  $\deg \mu = 2$ . Thus  $\mu : \hat{P} \rightarrow \mathbf{P}^2$  is a double cover with branch locus  $B_\mu$ . Assume that  $\alpha$  is the minimal even resolution of  $B_\mu$ , then  $\nu : \hat{P} \rightarrow Q$  is a smooth double cover with branch locus  $\tilde{B}_\mu$ .



# Conclusion

- InftyProject.
  - Research group of math information processing.
- Demo (*InftyReader*) to show the current state of the art.
- Adaptive method to improve character and symbol recognition (*CharImageManager*).
- Proposed some problems to be attacked.

Thanks you!

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Science Accessibility Net: <http://www.sciaccess.net/en/>