IMU/NAS WMDL symposium
Panel #3:
Data bases, digital libraries, encyclopediae

Washington, DC
June 1, 2012

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Wolfram Research, Inc.

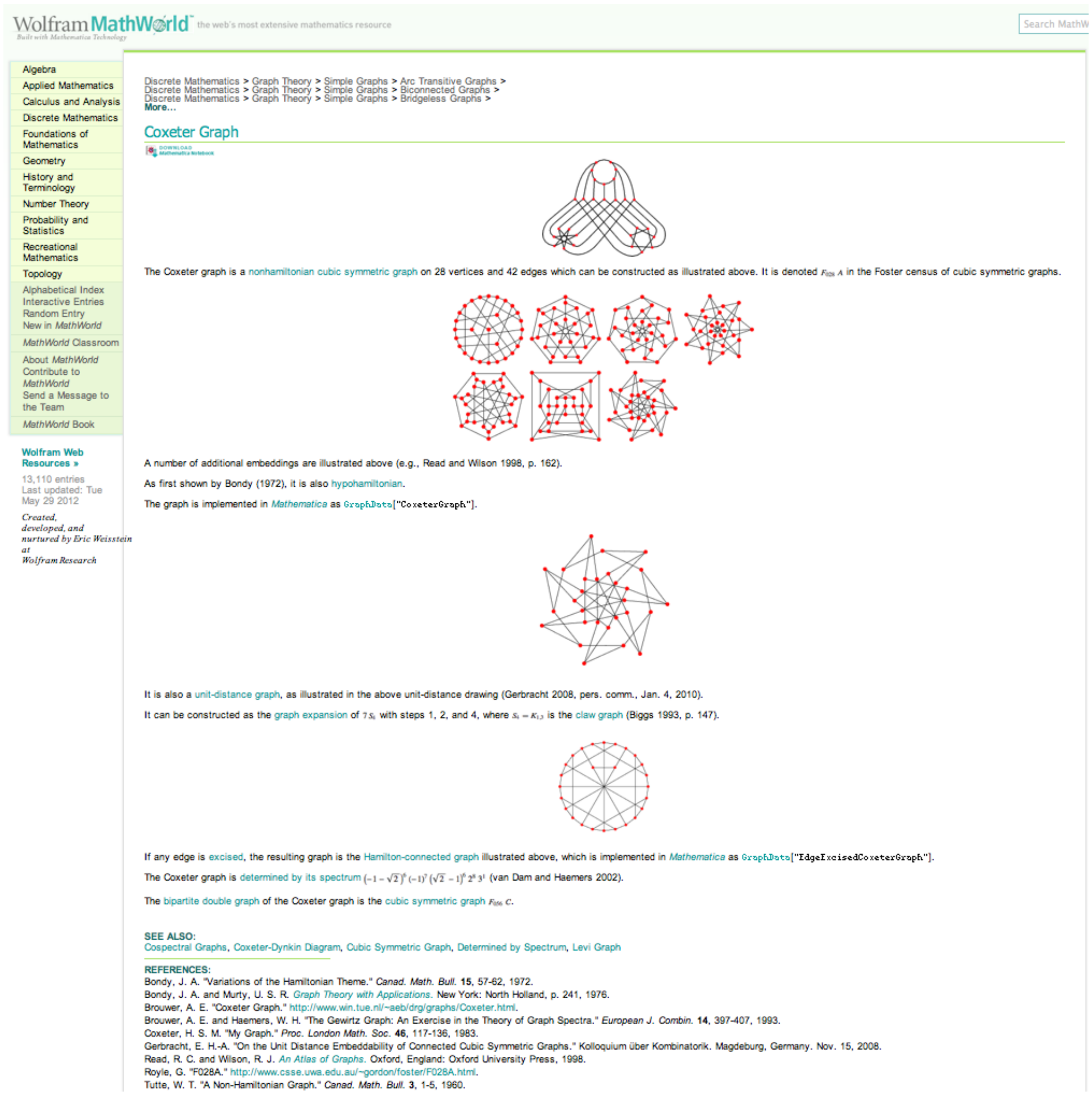
Data bases, digital libraries, encyclopediae: or -> a*nd!*

- Past and current: electronic mathematics resources generally fall into one (and only one) of the above three categories
- Goal for the future should be creation of new and integration of existing resources to allow more powerful, flexible, and universal access to mathematical information
- Requires tagging, interlinking, and integration with search and computation
- Tools and infrastructure are now becoming available -> exciting time to be working on digital libraries!

Some "typical" digital math encyclopediae: *MathWorld* and Wikipedia

- *MathWorld* is a free online encyclopedia of math including ~14,000 terms
  - Created and authored over ~20 years
  - First appeared on the internet in the mid-1990s
  - Content mainly textual + typeset math, tables, and figures
  - Uniform notations and conventions, but growth limited due to central editorial review
  - Contains literature citations but not direct linking
  - Much of the content makes use of computation (using *Mathematica*) and computational notebooks are available for download for most pages, but content itself displayed statically in HTML
- Wikipedia math content
  - Has existed for ~XX years, continues to grown
  - Does not use uniform notation and conventions
  - Some entries contain pseudocode, SVG figures, etc., but in general, cannot be considered "computable"
  - Often contains literature citations, frequently with direct linking

Example encyclopedia entry: *MathWorld*



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13,110 entries
Last updated: Tue May 29 2012

Created, developed, and nurtured by Eric Weisstein at Wolfram Research

Discrete Mathematics > Graph Theory > Simple Graphs > Arc Transitive Graphs >
Discrete Mathematics > Graph Theory > Simple Graphs > Biconnected Graphs >
Discrete Mathematics > Graph Theory > Simple Graphs > Bridgeless Graphs >
More...

Coxeter Graph

DOWNLOAD Mathematica Notebook

The Coxeter graph is a nonhamiltonian cubic symmetric graph on 28 vertices and 42 edges which can be constructed as illustrated above. It is denoted $F_{028}A$ in the Foster census of cubic symmetric graphs.

A number of additional embeddings are illustrated above (e.g., Read and Wilson 1998, p. 162).

As first shown by Bondy (1972), it is also hypohamiltonian.

The graph is implemented in *Mathematica* as GraphData["CoxeterGraph"].

It is also a unit-distance graph, as illustrated in the above unit-distance drawing (Gerbracht 2008, pers. comm., Jan. 4, 2010).

It can be constructed as the graph expansion of $7S_4$ with steps 1, 2, and 4, where $S_4 = K_{1,3}$ is the claw graph (Biggs 1993, p. 147).

If any edge is excised, the resulting graph is the Hamilton-connected graph illustrated above, which is implemented in *Mathematica* as GraphData["EdgeExcisedCoxeterGraph"].

The Coxeter graph is determined by its spectrum $(-1-\sqrt{2})^6 (-1)^7 (\sqrt{2}-1)^6 2^8 3^1$ (van Dam and Haemers 2002).

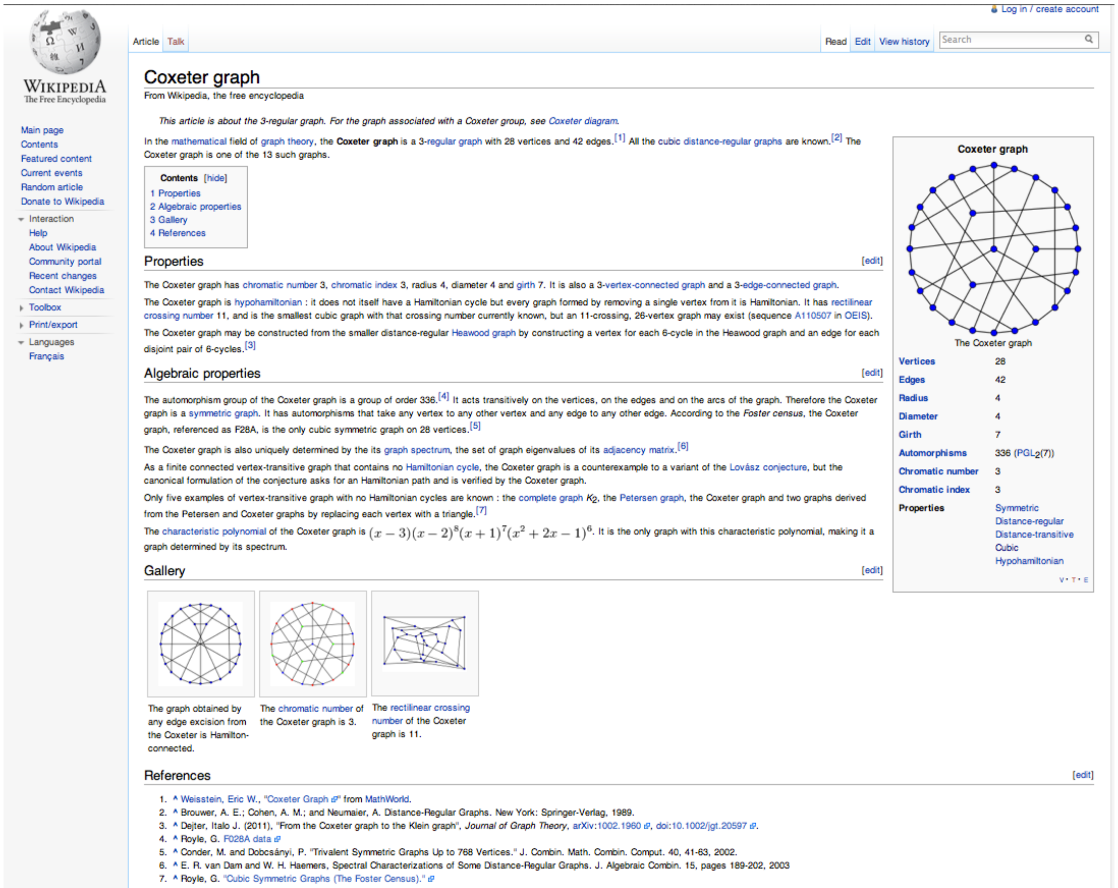
The bipartite double graph of the Coxeter graph is the cubic symmetric graph $F_{056}C$.

SEE ALSO:
Cospectral Graphs, Coxeter-Dynkin Diagram, Cubic Symmetric Graph, Determined by Spectrum, Levi Graph

REFERENCES:

Bondy, J. A. "Variations of the Hamiltonian Theme." *Canad. Math. Bull.* **15**, 57-62, 1972.
Bondy, J. A. and Murty, U. S. R. *Graph Theory with Applications.* New York: North Holland, p. 241, 1976.
Brouwer, A. E. "Coxeter Graph." http://www.win.tue.nl/~aeb/drg/graphs/Coxeter.html.
Brouwer, A. E. and Haemers, W. H. "The Gewirtz Graph: An Exercise in the Theory of Graph Spectra." *European J. Combin.* **14**, 397-407, 1993.
Coxeter, H. S. M. "My Graph." *Proc. London Math. Soc.* **46**, 117-136, 1983.
Gerbracht, E. H.-A. "On the Unit Distance Embeddability of Connected Cubic Symmetric Graphs." Kolloquium über Kombinatorik. Magdeburg, Germany. Nov. 15, 2008.
Read, R. C. and Wilson, R. J. *An Atlas of Graphs.* Oxford, England: Oxford University Press, 1998.
Royle, G. "F028A." http://www.csse.uwa.edu.au/~gordon/foster/F028A.html.
Tutte, W. T. "A Non-Hamiltonian Graph." *Canad. Math. Bull.* **3**, 1-5, 1960.

Example encyclopedia entry: Wikipedia



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Coxeter graph

From Wikipedia, the free encyclopedia

This article is about the 3-regular graph. For the graph associated with a Coxeter group, see Coxeter diagram.

In the mathematical field of graph theory, the **Coxeter graph** is a 3-regular graph with 28 vertices and 42 edges.[1] All the cubic distance-regular graphs are known.[2] The Coxeter graph is one of the 13 such graphs.

Contents [hide]

Coxeter graph

The Coxeter graph

Vertices	28
Edges	42
Radius	4
Diameter	4
Girth	7
Automorphisms	336 ($PGL_2(7)$)
Chromatic number	3
Chromatic index	3
Properties	Symmetric Distance-regular Distance-transitive Cubic Hypohamiltonian

V · T · E

Properties [edit]

The Coxeter graph has chromatic number 3, chromatic index 3, radius 4, diameter 4 and girth 7. It is also a 3-vertex-connected graph and a 3-edge-connected graph.

The Coxeter graph is hypohamiltonian : it does not itself have a Hamiltonian cycle but every graph formed by removing a single vertex from it is Hamiltonian. It has rectilinear crossing number 11, and is the smallest cubic graph with that crossing number currently known, but an 11-crossing, 26-vertex graph may exist (sequence A110507 in OEIS).

The Coxeter graph may be constructed from the smaller distance-regular Heawood graph by constructing a vertex for each 6-cycle in the Heawood graph and an edge for each disjoint pair of 6-cycles.[3]

Algebraic properties [edit]

The automorphism group of the Coxeter graph is a group of order 336.[4] It acts transitively on the vertices, on the edges and on the arcs of the graph. Therefore the Coxeter graph is a symmetric graph. It has automorphisms that take any vertex to any other vertex and any edge to any other edge. According to the *Foster census*, the Coxeter graph, referenced as F28A, is the only cubic symmetric graph on 28 vertices.[5]

The Coxeter graph is also uniquely determined by the its graph spectrum, the set of graph eigenvalues of its adjacency matrix.[6]

As a finite connected vertex-transitive graph that contains no Hamiltonian cycle, the Coxeter graph is a counterexample to a variant of the Lovász conjecture, but the canonical formulation of the conjecture asks for an Hamiltonian path and is verified by the Coxeter graph.

Only five examples of vertex-transitive graph with no Hamiltonian cycles are known : the complete graph K_2, the Petersen graph, the Coxeter graph and two graphs derived from the Petersen and Coxeter graphs by replacing each vertex with a triangle.[7]

The characteristic polynomial of the Coxeter graph is $(x-3)(x-2)^8(x+1)^7(x^2+2x-1)^6$. It is the only graph with this characteristic polynomial, making it a graph determined by its spectrum.

Gallery [edit]

The graph obtained by any edge excision from the Coxeter is Hamilton-connected.

The chromatic number of the Coxeter graph is 3.

The rectilinear crossing number of the Coxeter graph is 11.

References [edit]

1. ^ Weisstein, Eric W., "Coxeter Graph" from MathWorld.
2. ^ Brouwer, A. E.; Cohen, A. M.; and Neumaier, A. Distance-Regular Graphs. New York: Springer-Verlag, 1989.
3. ^ Dejter, Italo J. (2011), "From the Coxeter graph to the Klein graph", *Journal of Graph Theory*, arXiv:1002.1960, doi:10.1002/jgt.20597.
4. ^ Royle, G. F028A data
5. ^ Conder, M. and Dobcsányi, P. "Trivalent Symmetric Graphs Up to 768 Vertices." J. Combin. Math. Combin. Comput. 40, 41-63, 2002.
6. ^ E. R. van Dam and W. H. Haemers, Spectral Characterizations of Some Distance-Regular Graphs. J. Algebraic Combin. 15, pages 189-202, 2003
7. ^ Royle, G. "Cubic Symmetric Graphs (The Foster Census)."

Adding Computation: Wolfram|Alpha

- Converting classes of mathematical content into computable form (graphs, curves, surfaces, plane figures, finite groups, lattices, knots, ...)
- Started with the Computable Data Initiative at Wolfram Research for use in *Mathematica*
- Computable data now taken to another level with Wolfram|Alpha (free "computational knowledge engine" website)
- Mathematical objects are encoded in a custom database format
- Allows a combination of pre-computed properties for standard math objects and computation on arbitrary user-specified objects
- Allows unstructured (natural language) queries for properties of objects, a degree of interactivity via controls, export and import of math objects, exposure via custom APIs to arbitrary clients, direct exposure in *Mathematica*, chaining of results, ... (see Michael Trott's talk tomorrow for many more details and examples)

Wolfram|Alpha's take

WolframAlpha PRO

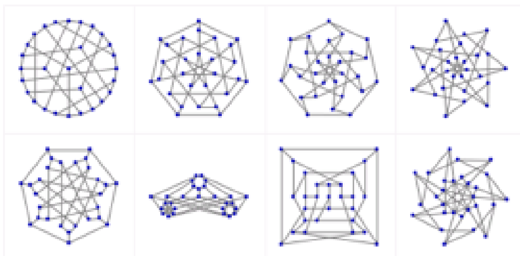
coxeter graph

Examples | Random

Input interpretation:

Coxeter graph

Images: | Single image



Alternate names:

11-crossing number graph A | 28-vertex transitive graph 9 | cubic transitive graph 53

Basic properties: | Show lists

vertex count	28
edge count	42
number of connected components	1

Graph features:

arc-transitive | biconnected | bridgeless | class 1 | connected | cubic | cyclic | determined by spectrum | distance-regular | distance-transitive | edge-transitive | hypohamiltonian | noncayley | noneulerian | nonhamiltonian | nonplanar | perfect matching | regular | square-free | symmetric | traceable | triangle-free | unit-distance | unitransitive | vertex-transitive | weakly regular

Vertex degrees:

3 (28 vertices)

Topological properties: | More properties

radius	4
diameter	4
girth	7
vertex connectivity	3
edge connectivity	3

Graph polynomials: | More polynomials

Characteristic polynomial:

$(x-3)(x-2)^8(x+1)^7(x^2+2x-1)^6$

Graph polynomials:

Fewer polynomials

Characteristic polynomial:

$(x-3)(x-2)^8(x+1)^7(x^2+2x-1)^6$

Idiosyncratic polynomial:

$(x+1)^7(x-24y-3)(x+3y-2)^8(x^2+2x-2y^2+4y-1)^6$

Independence polynomial:

$14x^{12}+336x^{11}+3752x^{10}+16660x^9+38556x^8+52928x^7+45836x^6+25788x^5+9506x^4+2268x^3+336x^2+28x+1$

Laplacian polynomial:

$(x-4)^7(x-1)^8x(x^2-8x+14)^6$

Matching polynomial:

$x^{28}-42x^{26}+777x^{24}-8344x^{22}+57708x^{20}-269640x^{18}+868700x^{16}-1934712x^{14}+2942247x^{12}-2970310x^{10}+1894851x^8-703080x^6+130872x^4-8904x^2+84$

Matching-generating polynomial:

$84x^{14}+8904x^{13}+130872x^{12}+703080x^{11}+1894851x^{10}+2970310x^9+2942247x^8+1934712x^7+868700x^6+269640x^5+57708x^4+8344x^3+777x^2+42x+1$

Coloring properties:

Chromatic number:

3

Edge chromatic number:

3

Spectrum:

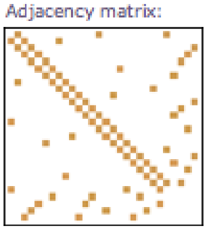
Approximate form

$(-1-\sqrt{2})^6(-1)^7(-1+\sqrt{2})^6 2^8 3^1$

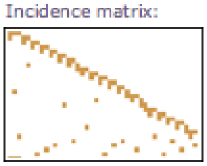
Associated matrices:

More matrices

Adjacency matrix:



Incidence matrix:



Graph indices:

More indices

Hosoya index	11 790 272
Kirchhoff index	369
stability index	11 388 416
Wiener index	1050

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Sources Download page

Drawbacks

- Exposition, literature citations, etc. are not smoothly integrated
- While it is possible (and has actually been done) to repackage *MathWorld'*s encyclopedic content for re-exposure as a Wolfram|Alpha database, the results are not fully computable (or at present interlinked)

WolframAlpha PRO

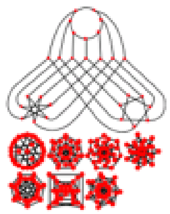
mathworld coxeter graph

Examples Random

Input interpretation:

Coxeter graph

Illustration:



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The bipartite double graph of the Coxeter graph is the cubic symmetric graph $F_{056}C$.

More information »

Related topics:

cospectral graphs | Coxeter-Dynkin diagram | cubic symmetric graph | determined by spectrum | Levi graph

Subject classifications: Show details

MathWorld:

arc transitive graphs | biconnected graphs | bridgeless graphs | class 1 graphs | connected graphs | cubic graphs | cyclic graphs | determined by spectrum graphs | distance-regular graphs | distance transitive graphs | edge-transitive graphs | hypohamiltonian graphs | noncayley graphs | noneulerian graphs | nonhamiltonian graphs | nonplanar graphs | perfect matching graphs | regular graphs | square-free graphs | symmetric graphs | traceable graphs | triangle-free graphs | unit-distance graphs | vertex-transitive graphs | weakly regular graphs

MSC 2010:

05Cxx

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The future: Computable, queriable, interlinked, integrated DML/-database/encyclopedia

- One prototype is Currently under development under a one-year grant from the Sloan Foundation to the Wolfram Foundation (PIs Michael Trott and Eric Weisstein)
- Prototype implementation is for a manageable (read: "small") subfield of mathematics of historical and practical interest: continued fractions
- Involves compilation, tagging, and presentation of identities, theorems, definitions, people involved, and literature

Prototype eCF implementation

- **Example early prototype theorem (raw markup)**

```
"SeidelSternTheorem" -> {
    Spellings -> Dev /@ {FO["seidel", Opt["-"], "stern", "theorem"]},
    "Name" -> "Seidel-Stern theorem",
    "ResultType" -> "Theorem",
    "WhatItIs" -> M["Tokens"->{f->L["Tokens"->{"ContinuedFraction"},
        "Specs"->{T0->M["Tokens"->{"Real"}]}],
        Z[n]->L["Tokens"->{"Sequence"},
        "Specs"->{X->M["Tokens"->{"Approximant"},"Specs"->{f->f}]}],
        Z[2n]->L["Tokens"->{"EvenSubsequence"},"Specs"->{b[n]->Z[n]}],
        Z[2n+1]->L["Tokens"->{"OddSubsequence"},"Specs"->{b[n]->Z[n]}]},
        "Givens"->{S[f,a[n]]->1,"ForAll"[n,S[f,b[n]]>0]},
        "Conclusions"->{M["Tokens" -> {Z[2n]},
        "Restrictions" -> {"SequenceConverges", "RealMonotonicity"}],
        M["Tokens" -> {Z[2n+1]},
        "Restrictions" -> {"SequenceConverges", "RealMonotonicity"}],
        ""}
    ],
    "Definition" -> Row[{"A positive continued fraction ", InlineForm["ContinuedFracti
    "DefinitionSource" -> "LorentzenWaadeland1992",
    "ConceptsInvolved" -> {"ContinuedFraction:Convergence"},
    "Provers" -> {"PhilippLudwigVonSeidel", "MoritzAbrahamStern"},
    "ProofDates" -> {1846, 1848},
    "Extensions" -> {"SeidelSternTheoremTransformed"},
    "References" -> {
        {"Seidel1846"},
        {"Stern1848"},
        {"LorentzenWaadeland1992", "Pages" -> 117, "Theorem" -> "3.13"},
        {"BeardonShort2010", "Theorem" -> "1.1"}
    }
}
```

Prototype eCF implementation

▪ Example early prototype theorem (default formatted result)

In[27]:= `Calculate["devmode:seidel-stern theorem"]`

Out[27]//TraditionalForm=

Assuming Seidel-Stern theorem | Use transformed Seidel-Stern theorem instead

Input interpretation:

Seidel-Stern theorem

Definition:

A positive continued fraction $\underset{n=1}{\overset{\infty}{\mathrm{K}}} 1/b_n$ converges if and only if $\sum_n b_n = \infty$. If $\sum_n b_n < \infty$ then $\underset{n=1}{\overset{\infty}{\mathrm{K}}} 1/b_n$ diverges generally.

(source: Lorentzen and Waadeland (1999))

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} \frac{a_k}{b_k}$ is a continued fraction

Relations:

Concepts involved:

continued fraction convergence

Extensions:

transformed Seidel-Stern theorem

History:

proof dates	1846 | 1848
provers	Philipp Ludwig von Seidel | Moritz Abraham Stern

References:

L. Seidel. "Untersuchungen über die Konvergenz und Divergenz der Kettenbrüche." Habilschrift. Munich, 1846.

M.A. Stern. "Über die Kennzeichen der Konvergenz eines Kettenbruchs." *Journal Für Die Reine Und Angewandte Mathematik* **37**, 255–272, 1848.

L. Lorentzen, H. Waadeland. Thm. 3.13 in *Continued Fractions with Applications*. Amsterdam: North-Holland Publishing Co., p. 117, 1992.

Alan F. Beardon, Ian Short. "The Seidel, Stern, Stolz and Van Vleck Theorems on Continued Fractions." *Bulletin Of The London Mathematical Society* **422**, 457–466, 2010.

Prototype eCF implementation

- **Another example early prototype theorem (default formatted result)**

```
In[3]:= Calculate["devmode:stern-stolz theorem"]
```

//TraditionalForm=

Input interpretation:

Stern-Stolz divergence theorem

Definition:

If $\sum_n |b_n| < \infty$, then the continued fraction $\underset{i=1}{\overset{\infty}{\mathrm{K}}} 1/b_i$ diverges generally, the sequences $\{A_{2n+m}\}_n$ and $\{B_{2n+m}\}_n$ converge absolutely to finite values $\mathcal{A}_m$ and $\mathcal{B}_m$ respectively (for $m = 0, 1$), and $\mathcal{A}_1 \mathcal{B}_0 - \mathcal{A}_0 \mathcal{B}_1 = 1$.

(source: Lorentzen and Waadeland (1999))

$|z|$ is the absolute value of z

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} \frac{a_k}{b_k}$ is a continued fraction

Relations:

Concepts involved:

generalized continued fraction | approximant

History:

proof dates	1848 | 1886
provers	Moritz Abraham Stern | Otto Stolz

References:

M.A. Stern. "Über die Kennzeichen der Konvergenz eines Kettenbruchs." *Journal Für Die Reine Und Angewandte Mathematik* **37**, 255–272, 1848.

M.A. Stern. *Lehrbuch der algebraischen Analysis.* Leipzig: Teubner, 1860.

O. Stolz. *Vorlesungen über allgemiene Arithmetic.* Leipzig: Teubner, 1886.

L. Lorentzen, H. Waadeland. *Continued Fractions with Applications.* Amsterdam: North-Holland Publishing Co., p. 100, 1992.

Douglas Bowman, James Mc Laughlin. Thm. 1 in *Asymptotics and Sequential Closures of Continued Fractions and their Generalizations.* p. 3, 2009.

Alan F. Beardon, Ian Short. "The Seidel, Stern, Stolz and Van Vleck Theorems on Continued Fractions." *Bulletin Of The London Mathematical Society* **422**, 457–466, 2010.

Prototype eCF implementation

▪ Named continued fraction example

In[31]:= `Calculate["devmode:rogers-ramanujan c.f."]`

Out[31]//TraditionalForm=

Input interpretation:

Rogers-Ramanujan continued fraction

Results:

Identity:

$$\frac{\sqrt[5]{q}\,(q;q^5)_\infty\,(q^4;q^5)_\infty}{(q^2;q^5)_\infty\,(q^3;q^5)_\infty} = \sqrt[5]{q}\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{q^{k-1}}{1}\right)$$

Convergence conditions:

$|q| < 1$

$(a;q)_n$ gives the q-Pochhammer symbol

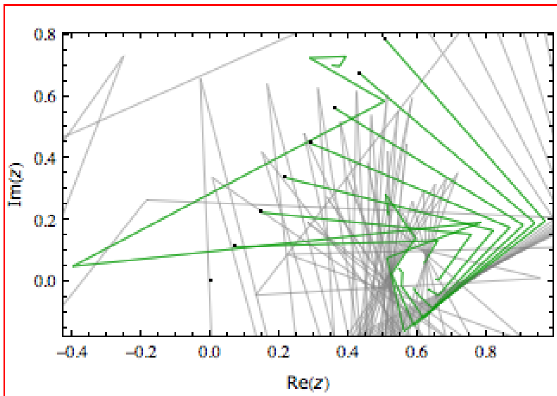
$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}}\frac{a_k}{b_k}$ is a continued fraction

$|z|$ is the absolute value of z

Convergents: More

$$\sqrt[5]{q},\ \sqrt[5]{q}\,\frac{1}{1+\frac{q}{1}},\ \sqrt[5]{q}\,\frac{1}{1+\frac{q}{1+\frac{q^2}{1}}},\ \ldots$$

Interactive plot:



(connected successive convergents starting from 16 points on a ray; converging convergents in green; initial z–points are shown as small black dots)

- ○ convergents as maps of the complex plane
- ◉ successive convergents in the complex plane
- ○ convergence of the convergents in the complex plane
- ○ convergence of the convergents at a single point

+ More controls

References:

L. J. Rogers. "Second Memoir on the Expansion of Certain Infinite Products." *Proceedings Of The London Mathematical Society* **25**, 318–343, 1894.

S. Ramanujan. *Notebooks, Vol. 1.* Mumbai: Tata Institute of Fundamental Research, 1957.

Prototype eCF implementation

- **Classes of continued fractions**

In[37]:= `Calculate["devmode:continued fraction identities involving arccosh"]`

Out[37]//TraditionalForm=

Input interpretation:

ArcCosh	continued fraction identities

Members:

| | | | | | (total: 7)

Results: Show conditions

$$\cosh^{-1}(z) = \frac{\sqrt{-1+z}\left(\frac{\pi}{2} - \frac{z\sqrt{1-z^2}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{-2\left(2\left\lfloor\frac{k+1}{2}\right\rfloor-1\right)\left\lfloor\frac{k+1}{2}\right\rfloor z^2}{2k+1}}\right)}{\sqrt{1-z}}$$

$$\cosh^{-1}(z) = \frac{z\sqrt{z^2-1}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\frac{k\left(k-(-1)^k\right)\left(-1+z^2\right)}{3+4(-1+k)(1+k)}}{1}}$$

$$\cosh^{-1}(z) = \frac{\sqrt{z^2-1}}{z\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{-\frac{k^2\left(z^2-1\right)}{z^2\left(4k^2-1\right)}}{1}\right)}$$

$$\frac{\cosh^{-1}(z)}{\sqrt{1-z^2}} = \frac{\sqrt{z-1}}{\sqrt{1-z}\left(z+\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{k^2\left(1-z^2\right)}{(2k+1)z}\right)}$$

$$\cosh^{-1}(z) = \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}} + \frac{z\sqrt{\frac{-1+z}{1+z}}}{(-1+z)\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\frac{k^2 z^2}{1-z^2}}{2k+1}\right)}$$

$$\cosh^{-1}(z) = \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}} - \frac{z\sqrt{-1+z}\sqrt{1+z}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{-\frac{k\left(-(-1)^k+k\right)z^2}{-1+4k^2}}{1}}$$

$$\cosh^{-1}(z) = \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}} - \frac{z\sqrt{-1+z}\sqrt{1+z}}{1-z^2+\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{k^2 z^2}{(2k+1)\left(1-z^2\right)^{\frac{1}{2}\left(1+(-1)^k\right)}}}$$

$\cosh^{-1}(x)$ is the inverse hyperbolic cosine function

$\lfloor x \rfloor$ is the floor function

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} \frac{a_k}{b_k}$ is a continued fraction

Prototype eCF implementation

- **Classes of continued fractions (with conditions via user controls)**

In[37]:= `Calculate["devmode:continued fraction identities involving arccosh"]`

t[37]//TraditionalForm=

Input interpretation:

ArcCosh	continued fraction identities

Members:

| | | | | | (total: 7)

Results: Hide conditions

$$\cosh^{-1}(z) = \frac{\sqrt{-1+z}\left(\frac{\pi}{2} - \frac{z\sqrt{1-z^2}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-2\left(2\left\lfloor\frac{k+1}{2}\right\rfloor-1\right)\left\lfloor\frac{k+1}{2}\right\rfloor z^2}{2k+1}}\right)}{\sqrt{1-z}}$$

for $(z \in \mathbb{C} \wedge -1 < z < 1) \vee (z \in \mathbb{C} \wedge z \notin \mathbb{R})$

$$\cosh^{-1}(z) = \frac{z\sqrt{z^2-1}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{k\left(k-(-1)^k\right)\left(-1+z^2\right)}{3+4(-1+k)(1+k)}}{1}}$$ for $z \in \mathbb{C} \wedge \mathrm{Re}(z) > 0 \wedge \neg (z \in \mathbb{R} \wedge 1 \leq z < \infty)$

$$\cosh^{-1}(z) = \frac{\sqrt{z^2-1}}{z\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\frac{k^2\left(z^2-1\right)}{z^2\left(4k^2-1\right)}}{1}\right)}$$ for

$z \in \mathbb{C} \bigwedge \mathrm{Re}(z) > 0 \bigwedge \neg (z \in \mathbb{R} \wedge 1 \leq z < \infty) \bigwedge \left|\arg\left(\frac{1}{z^2}\right)\right| < \pi$

$$\frac{\cosh^{-1}(z)}{\sqrt{1-z^2}} = \frac{\sqrt{z-1}}{\sqrt{1-z}\left(z+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{k^2\left(1-z^2\right)}{(2k+1)z}\right)}$$ for

$z \in \mathbb{C} \wedge \mathrm{Re}(z) > 0 \wedge \neg (z \in \mathbb{R} \wedge 1 \leq z < \infty) \wedge \left|\arg\left(1-z^2\right)\right| < \pi$

$$\cosh^{-1}(z) = \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}} + \frac{z\sqrt{\frac{-1+z}{1+z}}}{(-1+z)\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{k^2 z^2}{1-z^2}}{2k+1}\right)}$$ for $(z \in \mathbb{C} \wedge -1 < z < 1) \vee (z \in \mathbb{C} \wedge z \notin \mathbb{R})$

$$\cosh^{-1}(z) = \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}} - \frac{z\sqrt{-1+z}\sqrt{1+z}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\frac{k\left(-(-1)^k+k\right)z^2}{-1+4k^2}}{1}}$$ for $(z \in \mathbb{C} \wedge -1 < z < 1) \vee (z \in \mathbb{C} \wedge z \notin \mathbb{R})$

$$\cosh^{-1}(z) = \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}} - \frac{z\sqrt{-1+z}\sqrt{1+z}}{1-z^2+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{k^2 z^2}{(2k+1)\left(1-z^2\right)^{\frac{1}{2}\left(1+(-1)^k\right)}}}$$

for $(z \in \mathbb{C} \land -1 < z < 1) \lor (z \in \mathbb{C} \land z \notin \mathbb{R})$

$\cosh^{-1}(x)$ is the inverse hyperbolic cosine function

$\lfloor x \rfloor$ is the floor function

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} \frac{a_k}{b_k}$ is a continued fraction

$e_1 \land e_2 \land \ldots$ is the logical AND function

$e_1 \lor e_2 \lor \ldots$ is the logical OR function

$\mathrm{Re}(z)$ is the real part of z

$\neg$ expr is the logical NOT function

$\arg(z)$ is the complex argument

$|z|$ is the absolute value of z