

The 2010 Rolf Nevanlinna Prize

The winner of the 2010 Rolf Nevanlinna Prize is Daniel Spielman of Yale University USA. He has been awarded “*for smoothed analysis of Linear Programming, algorithms for graph-based codes and applications of graph theory for Numerical Computing.*”

Daniel Spielman’s Work

Linear Programming (LP) is one of the most useful tools in applied mathematics. It is basically a technique for the optimization of a objective function subject to linear equality and inequality constraints. Perhaps the oldest algorithm for LP (an algorithm is a finite sequence of instructions for solving a computational problem; it is like a computer programme) is what is known as the Simplex Method. The simplex algorithm was devised by George Dantzig way back in 1947 and is extremely popular for numerically solving linear programming problems even today. In geometric terms, the constraints define a convex polyhedron in a high-dimensional space, and the simplex algorithm reaches the optimum by moving from one vertex to a neighbouring vertex of the polyhedron.

The method works very well in practice even though in the worst case (on artificially constructed instances) the algorithm takes exponential time. Thus, understanding the complexity of LP and that of the simplex algorithm have been major problems in computer science. Mathematicians have been puzzled by the general efficacy of the simplex method in practice and have long tried to establish this as a mathematical theorem. While worst-case analysis is an effective tool to study the difficulty of a problem, it is not an effective tool to study the practical performance of an algorithm. An alternative is the average case analysis, where the performance is averaged over all instances of a given size, but, real instances of the problem arising in practice, may not be the same as average case instances. So average case analysis is not always appropriate.

In 1979 L. G. Kachian proved that LP is in P; that is, there is a polynomial time algorithm for linear programming and this led to the discovery of polynomial algorithms for many optimization problems. The general belief then was that there was a genuine tradeoff between being good in theory and being good in practice; that these two may not coexist for LP. However, Narendra Karmarkar’s interior point method in 1984 and subsequent variants thereof, whose complexity is polynomial, shattered this belief. Interior point methods construct a sequence of feasible points, which lie in the interior of the polyhedron but never on its boundary (as against the vertex-to-vertex path of the simplex algorithm), that converges to the solution. Karmarkar’s algorithm and the later theoretical and practical discoveries are competitive with, and occasionally superior to, the simplex method.

But in spite of these developments, the simplex method remains the most popular method for solving LP problems and there was no satisfactory explanation for its excellent performance till the beautiful concept of smoothed analysis introduced by Spielman and Shang-Hua Teng enabled them to prove a mathematical theorem.

Smoothed analysis gives a more realistic analysis of the practical performance of the algorithm than using worst-case or average case scenarios. It provides “a means,” according to Spielman, “of explaining the practical success of algorithms and heuristics that have poor worst-case behaviour,” Smoothed analysis is a hybrid of worst-case and average-case analysis that inherits the advantages of both. “Spielman and Teng’s proof is really a *tour de force*,” says Gil Kalai, a professor of mathematics at the Hebrew University in Jerusalem and an adjunct professor of mathematics and computer science at Yale.

The “smoothed complexity” of an algorithm is given by the performance of the algorithm under slight random perturbations of the worst-case inputs. If the smoothed complexity is low, then the simplex algorithm, for example, should perform well in practical cases where input data are subject to noise and perturbations. That is, while there may be pathological examples where the method fails, slight modifications of any pathological example yields a “smooth” problem on which the Simplex Method works very well. “Through smoothed analysis, theorists may find ways to appreciate heuristics they may have previously rejected. Moreover, we hope that smoothed analysis may inspire the design of successful algorithms that might have been rejected for having poor worst-case complexity,” Spielman has said.

The Association for Computing Machinery (ACM) and the European Association for Theoretical Computer Science (EATCS) awarded the 2008 Gödel Prize to Spielman and Teng for developing the tool. Spielman and Teng’s paper on ‘*Smoothed analysis of algorithms: Why the simplex algorithm usually takes polynomial time*’ in the Journal of the ACM was also one of the three winners of the 2009 Fulkerson Prize awarded jointly by the American Mathematical Society (AMS) and the Mathematical Programming Society (MPS).

Since its introduction in 2001, smoothed analysis has been used as a basis for considerable research, for problems ranging from mathematical programming, numerical analysis, machine learning and data mining. “However,” writes Spielman, “we still do not know if Smoothed Analysis, the most ambitious and theoretical of my analyses, will lead to improvements in practice.”

A second major contribution of Spielman is in the area of coding. Much of the present day communication uses coding, either for preserving secrecy or for ensuring error-free communication. Error-correcting codes (ECCs) are the means by which interference in communication is compensated. ECCs play a significant role in modern technology, from satellite communications to computer memories. In ECC, one can recover the correct message at the receiving end, even if a number of

bits of the message got corrupted, provided the number is below a threshold. Spielman's work has aimed at developing codes that are quickly encodable and decodable, and which allow communication at rates approaching the capacity of the communication channel.

In information theory low-density parity-check code (LDPC) is a linear ECC, which corrects on a block-by-block basis and enables message transmission over a noisy channel. LDPC codes are constructed using certain class of sparse graphs (where the number of edges is much less than the possible number of edges). They are capacity approaching codes, which means that practical constructions exist that allow noise threshold to be set very close to the limit of what is theoretically possible. Using certain iterative techniques, LDPC codes can be decoded in time linear to their block length.

An important technique to make both coding and decoding efficient is based on extremely well-connected, but sparse, graphs called 'expanders'. It is this counterintuitive and apparently contradictory feature of expanders – that they are sparse and yet highly connected – that makes expanders very useful, points out Peter Sarnak, a professor of mathematics at Princeton University. Spielman and his co-authors have done fundamental work using such graph theoretic methods and have designed very efficient methods for coding and decoding.

The most famous of Spielman's papers in this field was '*Improved Low-density Parity Check Codes Using Irregular Graphs*', which shared the 2002 Information Theory Society Paper Award. (Regular graphs, as opposed to irregular graphs, where each vertex has the same number of neighbours.) This paper demonstrated that irregular low-density parity check codes (LDPCs) could perform much better than the more common ones that use regular graphs on the additive white Gaussian noise (AWGN) channel.

This was an extension of the earlier work on '*Efficient Erasure Correcting Codes*' by Spielman and others, which introduced irregular LDPC codes and proved that they could approach the capacity of the so-called Binary Erasure Channel. So these codes provide an efficient solution to problems such as packet-loss over the internet and are particularly useful in multicast communications. They also provide one of the best known coding techniques for minimizing power consumption required to achieve reliable communication in the presence of white Gaussian noise. Irregular LDPC codes have had many other applications, including the recent DVB-S2 (Digital Video Broadcasting – Satellite v2) standard.

Spielman has applied for five patents for ECCs that he has invented and four of them have already been granted by the U. S. patent Office.

Combinatorial scientific computing is the name given to the interdisciplinary field in which one applies graph theory and combinatorial algorithms to problems in computational science and engineering. Spielman has recently turned his attention

to one of the most fundamental problems in computing: the problem of solving a system of linear equations, which is central to scientific and engineering applications, mathematical programming and machine learning. He has found remarkably linear time algorithms based on graph partitioning for several important classes of linear systems. These have led to considerable theoretical advances as well as to practically good algorithms.

“The beautiful interface between theory and practice, be it in mathematical programming, error-correcting codes, the search for sparsifiers meeting the so-called Ramanujan bound, analysis of algorithms, computational complexity theory or numerical analysis, is characteristic of Dan Spielman’s work,” says Kalai.

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