

Mathematics in Ancient India

Highly intellectual and given to abstract thinking as they were, one would expect the ancient Indians to excel in mathematics. Europe got its early arithmetic and algebra from the Arabs—hence the ‘Arabic numerals’—but the Arabs themselves had previously taken them from India. The astonishing progress that the Indians had made in mathematics is now well known and it is recognized that the foundations of modern arithmetic and algebra were laid long ago in India. The clumsy method of using a counting frame and the use of Roman and such like numerals had long retarded progress when the ten Indian numerals, including the zero sign, liberated the human mind from these restrictions and threw a flood of light on the behavior of numbers. These number symbols were unique and entirely different from all other symbols that had been in use in other countries. They are common enough today and we take them for granted, yet they contained the germs of revolutionary progress in them. It took many centuries for them to travel from India, via Baghdad, to the western world.

A hundred and fifty years ago, during Napoleon’s time, Laplace wrote:

‘It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position, as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit, but its very simplicity, the great ease which it has lent to all computations puts our arithmetic in the first rank of useful inventions; and we shall appreciate the grandeur of this achievement when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity.’*

The origins of geometry, arithmetic, and algebra in India go back to remote periods. Probably to begin with there was some kind of geometrical algebra used for making figures for Vedic altars. Mention is made in the most ancient books of the geometrical method for the transformation of a square into a rectangle having a given side: $ax = c$. Geometrical figures are even now commonly used in Hindu ceremonies. Geometry made progress in India but in this respect Greece and Alexandria went ahead. It was in arithmetic and algebra that India kept the lead. The inventor or inventors of

*Quoted in Hogben’s *Mathematics for the Million*, London, 1942.

the decimal place-value system and the zero mark are not known. The earliest use of the zero symbol so far discovered, is in one of the scriptural books dated about 200 BC. It is considered probable that the place-value system was invented about the beginning of the Christian era. The zero, called *shunya* or nothing was originally a dot and later it became a small circle. It was considered a number like any other. Professor Halsted thus emphasizes the vital significance of this invention: 'The importance of the creation of the zero mark can never be exaggerated. This giving to airy nothing, not merely a local habitation and a name, a picture, a symbol but helpful power, is the characteristic of the Hindu race from whence it sprang. It is like coining the *Nirvana* into dynamos. No single mathematical creation has been more potent for the general on-go of intelligence and power.'^{*}

Yet another modern mathematician has grown eloquent over this historic event. Dantzig in his *Number* writes:

'This long period of nearly five thousand years saw the rise and fall of many a civilization, each leaving behind it a heritage of literature, art, philosophy, and religion. But what was the net achievement in the field of reckoning, the earliest art practised by man? An inflexible numeration so crude as to make progress well nigh impossible, and a calculating device so limited in scope that even elementary calculations called for the services of an expert.... Man used these devices for thousands of years without making a single worthwhile improvement in the instrument, without contributing a single important idea to the system.... Even when compared with the slow growth of ideas during the dark ages, the history of reckoning presents a peculiar picture of desolate stagnation. When viewed in this light the achievements of the unknown Hindu, who sometime in the first centuries of our era discovered the principle of position, assumes the importance of a world event.'[†]

Dantzig is puzzled at the fact that the great mathematicians of Greece did not stumble on this discovery. 'Is it that the Greeks had such a marked contempt for applied science, leaving even the instruction of their children to slaves? But if so, how is it that the nation that gave us geometry and carried this science so far did not create even a rudimentary algebra? Is it not equally strange that algebra, that corner-stone of modern mathematics, also originated in India, and at about the same time that positional numeration did?'

^{*}G.B. Halsted, *On the Foundation and Technique of Arithmetic*, p. 20, Chicago, 1912, quoted in *History of Hindu Mathematics*, B. Datta and A.N. Singh, 1935.

[†] Quoted in L. Hogben's '*Mathematics for the Million*' (London, 1942).

The answer to this question is suggested by Professor Hogben:

‘The difficulty of understanding why it should have been the Hindus who took this step, why it was not taken by the mathematicians of antiquity, why it should first have been taken by practical man, is only insuperable if we seek for the explanation of intellectual progress in the genius of a few gifted individuals, instead of in the whole social framework of custom thought which circumscribes the greatest individual genius. What happened in India about AD 100 had happened before. May be it is happening now in Soviet Russia.... To accept it (this truth) is to recognise that every culture contains within itself its own doom, unless it pays as much attention to the education of the mass of mankind as to the education of the exceptionally gifted people.’*

We must assume then that these momentous inventions were not just due to the momentary illumination of an erratic genius, much in advance of his time, but that they were essentially the product of the social *milieu* and that they answered some insistent demand of the times. Genius of the high order was certainly necessary to find this out and fulfil the demand, but if the demand had not been there the urge to find some way out would have been absent, and even if the invention had been made it would have been forgotten or put aside till circumstances more propitious for its use arose. It seems clear from the early Sanskrit works on mathematics that the demand was there, for these books are full of problems of trade and social relationship involving complicated calculations. There are problems dealing with taxation, debt, and interest; problems of partnership, barter and exchange, and the calculation of the fineness of gold. Society had grown complex and large numbers of people were engaged in governmental operations and in an extensive trade. It was impossible to carry on without simple methods of calculation.

The adoption of zero and the decimal place-value system in India unbarred the gates of the mind to rapid progress in arithmetic and algebra. Fractions come in, and the multiplication and division of fractions; the rule of three is discovered and perfected; squares and square-roots (together with the sign of the square root, $\sqrt{\quad}$); cubes and cube roots; the minus sign; tables of sines; π is evaluated as 3.1416; letters of the alphabet are used in algebra to denote unknowns; simple and quadratic equations are considered; the mathematics of zero are investigated. Zero is defined as $a - a = 0$; $a + 0 = a$; $a - 0 = a$; $a \times 0 = 0$; $a \div 0$ becomes infinity. The conception of negative quantities also comes in, thus: $\sqrt{4} = +2$.

These and other advances in mathematics are contained in books written by a succession of eminent mathematicians from the fifth to the

*Hogben: ‘*Mathematics for the Million*’ (London, 1942), p. 285.

twelfth century AC. There are earlier books also (Baudhayana, c. eighth century BC; Apastamba and Katyayana, both c.fifth century BC) which deal with geometrical problems, especially with triangles, rectangles, and squares. But the earliest extant book on algebra is by the famous astronomer, Aryabhata, who was born in AC 476. He wrote this book on astronomy and mathematics when he was only twenty-three years old. Aryabhata, who is sometimes called the inventor of algebra, must have relied, partly at least, on the work of his predecessors. The next great name in Indian mathematics is that of Bhaskara 1 (AC 522), and he was followed by Brahmagupta (AC 628), who was also a famous astronomer, and who stated the laws applying to *shunya* or zero and made other notable advances. There followed a succession of mathematicians who have written on arithmetic or algebra. The last great name is that of Bhaskara II, who was born in AC 1114. He wrote three books, on astronomy, algebra, and arithmetic. His book on arithmetic is known as 'Lilavati', which is an odd name for a treatise on mathematics, as it is the name of a woman. There are frequent references in the book to a young girl who is addressed as 'O Lilavati' and is then instructed on the problems stated. It is believed, without any definite proof, that Lilavati was Bhaskara's daughter. The style of the book is clear and simple and suitable for young persons to understand. The book is still used, partly for its style, in Sanskrit schools.

Books on mathematics continued to appear (Narayana in 1150, Ganesha in 1545), but these are mere repetitions of what had been done. Very little original work on mathematics was done in India after the twelfth century till we reach the modern age.

In the eighth century, during the reign of Khalif Al-Mansur (753-774), a number of Indian scholars went to Baghdad and among the books they took with them were works on mathematics and astronomy. Probably even earlier than this, Indian numerals had reached Baghdad, but this was the first systematic approach, and Aryabhata's and other books were translated into Arabic. They influenced the development of mathematics and astronomy in the Arab world, and Indian numerals were introduced. Baghdad was then a great centre of learning and Greek and Jewish scholars had gathered there bringing with them Greek philosophy, geometry and science. The cultural influence of Baghdad was felt throughout the Muslim world from Central Asia to Spain, and a knowledge of Indian mathematics in their Arabic translations spread all over this vast area. The numerals were called by the Arabs 'figures of Hind' (or India), and the Arabic word for a number is 'Hindsah', meaning 'from Hind'.

From this Arab world the new mathematics travelled to European countries, probably through the Moorish universities of Spain, and became the foundation for European mathematics. There was opposition in Europe to the use of the new numbers, as they were considered infidel symbols, and it took several hundred years before they were in common use. The earliest known use is in a Sicilian coin of 1134; in Britain the first use is in 1490.

It seems clear that some knowledge of Indian mathematics, and especially of the place-value system of numbers, had penetrated into Western Asia even before the formal embassy carried books to Baghdad. There is an interesting passage in a complaint made by a Syrian scholar-monk who was hurt at the arrogance of some Greek scholars who looked down on Syrians. Severus Sebokht was his name, and he lived in a convent situated on the Euphrates. He writes in AC 662 and tries to show that the Syrians were in no way inferior to the Greeks. By way of illustration he refers to the Indians: 'I will omit all discussion of the science of the Hindus, a people not the same as the Syrians; their subtle discoveries in the science of astronomy, discoveries that are more ingenious than those of the Greeks and the Babylonians; their computing that surpasses description. I wish only to say that this computation is done by means of nine signs. If those who believe, because they speak Greek, that they have reached the limits of science, should know of these things, they would be convinced that there are also others who know something.'*

Mathematics in India inevitably makes one think of one extra-ordinary figure of recent times. This was Srinivasa Ramanujam. Born in a poor Brahmin family in South India, having no opportunities for a proper education, he became a clerk in the Madras Port Trust. But he was bubbling over with some irrepressible quality of instinctive genius and played about with numbers and equations in his spare time. By a lucky chance he attracted the attention of a mathematician who sent some of his amateur work to Cambridge in England. People there were impressed and a scholarship was arranged for him. So he left his clerk's job and went to Cambridge and during a very brief period there did work of profound value and amazing originality. The Royal Society of England went rather out of their way and made him a Fellow, but he died two years later, probably of tuberculosis, at the age of thirty-three. Professor Julian Huxley has, I believe, referred to him somewhere as the greatest mathematician of the century.

Ramanujam's brief life and death are symbolic of conditions in India. Of our millions how few get any education at all, how many live on the verge of starvation; of even those who get some education how many have nothing to look forward to but a clerkship in some office on a pay that is usually far less than the unemployment dole in England. If life opened its gates to them and offered them food and healthy conditions of living and education and opportunities of growth, how many among these millions would be eminent scientists, educationists, technicians, industrialists, writers and artists, helping to build a new India and a new world?

*Quoted in *History of Hindu Mathematics*, by B. Datta and A. N. Singh, 1933. I am indebted to this book for much information on this subject.