Interview with Terence Tao

"At age two I tried teaching other kinds to count using number blocks"

Terence Tao (Adelaide, 1975) was just 13 years old when he won the gold medal at the International Mathematical Olympiad. In the two previous editions he had won the bronze and silver medal. He is currently full professor at the University of California in Los Angeles. He has received prestigious prizes, such as the Salem prize in 2000 and the Clay Foundation award in 2003. In this interview he encourages the reader "to play with mathematics", and comments on the 'public image' of mathematics; the way they are portrayed in the movies, for instance: "Very few of them give anything close to an accurate perception of what mathematics is, and what it is like to do it", says Tao.

You were a very young winner of the International Mathematical Olympiads. How did you get interested in Mathematics? Would you say it was something innate or did it also have to do with a particularly good teacher, for instance?

My parents tell me I was fascinated by numbers even at age two, when I tried teaching other kinds to count using number blocks. I remember as a child being fascinated with the patterns and puzzles of mathematical symbol manipulation. It wasn't until somewhat later, in college, that I also began to appreciate the meaning and purpose behind mathematics, and how it connects with the real world and with one's own intuition. Actually, I enjoy this deeper level of mathematics now much more than the problem-solving or symbolic aspects.

I think the most important thing for developing an interest in mathematics is to have the ability and the freedom to play with mathematics – to set little challenges for oneself, to devise little games, and so on. Having good mentors was very important for me, because it gave me the chance to discuss these sorts of mathematical recreations; the formal classroom environment is of course best for learning theory and applications, and for appreciating the subject as a whole, but it isn't a good place to learn how to experiment. Perhaps one character trait which does help is the ability to focus, and perhaps to be a little stubborn. If I learned something in class that I only partly understood, I wasn't satisfied until I was able to work the whole thing out; it would bother me that the explanation wasn't clicking together like it should. So I'd often spend a lot of time on very simple things until I could understand them backwards and forwards, which really helps when one then moves on to more advanced parts of the subject.

How do you look for new problems to work with? And how do you know a particular problem will be really interesting?

I pick up a lot of problems (and collaborators) by talking to other mathematicians. I was perhaps lucky that my original field, harmonic analysis, has so many connections and applications to other areas of mathematics (PDE, applied mathematics, number theory, combinatorics, ergodic theory, etc.), so there was never any shortage of problems to work on. Sometimes I can stumble across an interesting problem by systematically surveying a certain field and then discovering a gap in the literature; for instance, by taking an analogy between two different objects (e.g. two different PDE) and comparing the known positive and negative results for both.

There are some vague and general guestions which I would like to pursue (e.g. "How to control the long-time dynamics of evolution equations?"; "What is the best way to separate structure from randomness in combinatorial problems?"). I'm drawn to problems which, while offering some promise of progress in one of these questions, preferably by forcing one to develop a new technique, is also placed in as simple a setting as possible (a "toy model"), where all but one of the difficulties has been "turned off". Of course, it is often not obvious a priori what the difficulties will be, although this seems to be easier to work out with experience. I'm also a great fan of interdisciplinary research - taking ideas and insights from one field and applying them to another. For instance, my work with Ben Green on progressions in the primes came in part from my trying to understand the ideas behind Furstenberg's ergodic theory proof of Szemeredi's theorem, which turned out to be very compatible with the number-theoretic and Fourier-analytic arguments that Ben had in mind for this problem.

Are there such things as 'hot topics' in Mathematics? If so, which would you say are the hot topics now?

I am only really familiar with the areas of mathematics that I actively work on, so I cannot say what are the 'hot' things in other fields. But in my own fields, it seems that nonlinear geometric PDE is taking off right now (most dramatically in Perelman's use of the Ricci flow to solve the Poincaré conjecture) - there is an increasingly exciting synthesis between geometric, analytic, topological, dynamical, and algebraic methods here. The combinatorial approach to number theory, in which one develops results on specific sets (such as primes) by first establishing results involving much more arbitrary sets (e.g. sets of integers of positive density), also is rather active right now, and promises to offer a rather different set of tools (including ergodic theory!) to the other methods we currently have in analytic number theory.

What would you say the relationship is between Mathematics and the general public? How should it be ideally?

It probably varies quite a bit from country to country. In the United States, there seems to be a vague consensus among the public that mathematics is somehow "important" for various high-technology industries, but is also "hard", and best left to experts. So there is support for funding mathematical research, but not much interest in finding out exactly what it is that mathematicians do. (There have been a recent spate of films and other media involving mathematicians, but unfortunately very few of them give anything close to an accurate perception of what mathematics is, and what it is like to do it). I'd like to see mathematics demystified more, and to be made more accessible to the public, though I am not really sure how to try to achieve these goals.